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TRANSACTIONS

OF THE

AMERICAN INSTITUTE

OF

ELECTRICAL ENGINEERS.

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AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

New York, October 26th, 1898.

The 128th meeting of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS was held at 12 West 31st Street this date and was called to order by President Kennelly, at 8.10 P. M.

THE SECRETARY:—At the meeting of the Executive Committee this afternoon, the following Associated Members were elected:

Name.	Address.	Endorsed by
MURPHY, JOHN MOL.	Electrical Engineer, Safety Third Rail Electric Co., 5 Beekman St., Room 233 N. Y.	G. T. Hanchett. R. V. Baillard. Ralph W. Pope.
VREELAND, F. K.	2nd Ass't. Engineer, Crocker-Wheeler Electric Co., residence, 298 Orange Road, Montclair, N. J.	Henry Morton. F. B. Crocker. Gano S. Dunn.
WOODBRIDGE, J. E.	Editor *Electrical World* 9 Murray Street, N. Y.	T. R. Taltavall. A. E. Kennelly. Townsend Wolcott

Total 3

The following Associate Members were transferred to full membership:

Approved by Board of Examiners, Sept. 22d, 1898.

C. H. WORDINGHAM	City Electrical Engineer, Manchester, England.
WILLIAM STANLEY,	Electrical Engineer and Inventor, Pittsfield, Mass.

In the President's Inaugural Address at Omaha, a recommendation was made by him that the INSTITUTE undertake to bring about a plan of cooperation with the various colleges in regard to experimental research work. In accordance with that recommendation, the Council passed a resolution that a com-

mittee of seven should be appointed, and the President will inform you as to the action that was taken by that Committee to-day, and the progress of the work.

THE PRESIDENT:—I may say that the Committee has held its first meeting and a circular letter is being prepared to send to the various technical colleges and laboratories throughout the country, inviting their cooperation with the INSTITUTE, and submitting to them a list of problems in connection with practical subjects upon which measurements, observations and data are required. Sufficient encouragement has already been received from a number of college laboratories, to ensure the initiation of this movement, and it is hoped that all the principal technical institutions will take the matter up. The plan presents advantages to all parties concerned. The instructors receive information as to the practical work calling for observation and measurement from those engaged in electrical engineering. The students are stimulated to their best efforts by feeling that the work upon which they are engaged is of direct practical application. The INSTITUTE gains by the publication of the results obtained, as well as by the reception of material for good papers. It is not expected that the results of such investigations should be published exclusively or voluminously in the TRANSACTIONS. The main results of each investigation with the names of those engaged upon it, would be, as a general rule, all that the members of the INSTITUTE desire to obtain. In this way no extra expense is attached either to the INSTITUTE or to the technical colleges engaged in the experimental work.

It is desirable that the members of the INSTITUTE should communicate to the Secretary such problems as may arise in their practical work and which call for experimental investigation. The Committee would receive these suggested problems and add them to the list of material for investigation.

The paper before the meeting this evening is "An Electrical Survey in the Borough of Manhattan, New York City," and Mr. Knudson will now read it to us.

[The author read the paper. See page 653, October issue.]

Discussion.

Mr. William Mayer, Jr.:—I feel that the Institute is under great obligations to Mr. Knudson for presenting to us to-night in such an admirably succinct and clear manner the more important results of his electrical survey of the city; a survey which the text shows has extended over several years, and which has entailed a large number of tests, many of which were made at all hours of the night and day. And necessarily so, for it is obvious that to arrive at a definite conclusion as to the source of the fluctuating currents in the vicinity of 135th street, the effects must be observed when the known and possible sources were temporarily absent. In this case Mr. Knudson was rewarded by discovering a normally suppressed source, namely, the comparatively steady current from the incandescent light circuits.

The exhibits which Mr. Knudson has been at the trouble to procure for us are excellent instances of the biter bitten. If the state of affairs which these exhibits illustrate may be taken as at all comparable with what is taking place at the rails and tie-rods of similar roads all over the country, it should teach the managers of such roads that it may be a matter of economy to provide efficient conductors for the return current apart from gas and water pipes, etc.

The tie-rod which Mr. Knudson exhibits appears to have corroded to a greater extent in the middle, and towards the middle than at the ends. Inasmuch as it may be assumed that all parts of the rod were at equal potentials, and that the soil between the rails was of a uniform character, I am somewhat at a loss to account for this action. Perhaps Mr. Knudson or some other member may be able to offer an explanation, which doubtless will be interesting.

Since learning of Mr. Knudson's intention of presenting this paper, I have taken occasion to make inquiry of some of the gas companies as to whether any electrolytic effect had been thus far observed in the vicinity of 135th street, and was informed that a recent careful examination of the pipes had not disclosed any such action; which is so far reassuring, and also is confirmatory of Mr. Knudson's tests. It does not, however, prove that some such action may not be taking place at a remoter point where the current leaves the pipes.

In connection with my duties as supervising electrician of the high-tension subways in this city, I have naturally been on the alert for any evidences of electrolysis that might arise. I may repeat what I have on other occasions stated in this regard, namely, that no clear evidence of electrolysis has thus far been discovered on the cables or iron ducts of the subways of this city. I should perhaps make the further statement which, however, is known to most of you, that until comparatively re-

cently there may have been no electric traction companies, either overhead trolley or open conduit in New York city, proper. There have been one or two instances where corrosion of cables has been observed, but investigation indicated that it was due to chemical action pure and simple.

What was said to be a case of electrolysis in a "solid" iron tube which passed through one of the hand boxes of the subways on Park Row, this city, was reported to me four or five years ago. On investigation I found a small hole about three-fourths of an inch in diameter in the said tube. As, however, I was aware that there had been a "burn out" in a joint of one of the "high-tension" cables in close proximity to this pipe I concluded that this was the result of a different kind of electrolysis to that which we are discussing. Furthermore, a test of the conductors in the tube in question showed them to be up to the usual standard; proving that the defect in the tube was of very recent origin; the insulating material of the conductors being of a fibrous nature.

I mention this unusual incident, however, because of the fact that at the time of its occurrence I made an electrical survey of this entire neighborhood, over a distance of about half a mile from the Post Office east, testing from the subways to gas and water mains, and elevated road structures, and obtained readings of from two-tenths to five-tenths of a volt at every point tested. The deflections were steady, indicating the source to be at some electric light station. These tests were made, it will be understood, long before the trolley roads crossed the Brooklyn bridge. As a result I concluded that readings of this nature would probably be found in any city where electric light and power, or even telegraph terminal stations were in operation.

Mr. Knudson's query at the bottom of page 663 regarding the possible condition of the anchor bolts and iron foundations of the pillars of the L structure where his tests showed them to be positive to the surface rails and water pipes, recalls to my mind a case in which I was interested some time ago. It does not answer his inquiry, but it may be interesting in connection with it. At the time when the surface roads of Brooklyn had decided to change from horse power to the overhead trolley system, application was made by one of the surface roads to one of the elevated roads in that city for permission to use their structure for the support of their trolley wires and feeders, wherever they ran under them, which, as is well known, is the case for considerable distances. The chief concern of the elevated road authorities was as to what effect a possible contact of the trolley wire with their structure would have upon a passenger on their lines, or upon a passer-by in the street, who might, at that time, touch the structure. Being consulted on that and other points, I expressed the opinion that no apprehension need be felt on the score of injury to any one coming in contact with the structure

under the conditions stated, excepting under almost impossible circumstances, but I strongly recommended for other reasons that in the event of the desired permission being given, it be provided that no intentional electrical contact should ever be made with the elevated structure. The desired permission was given, but I presume the recommendation was overlooked, for in the course of a year or two it was discovered that the surface road had at various places connected their rails by large copper wires to the iron pillars of the structure, under the surface. On being required to vacate the premises, so to speak, I believe it was naively claimed that this use of the structure went with the right to suspend their wires from the structure. If I am not mistaken, I think the further contention was made that the passage of the electric current through the structure would be a positive benefit to it; possibly on the principle that "electricity is life." Of course the structure was an excellent return wire.

The information which Mr. Knudson has been at the trouble to obtain for us regarding the Brooklyn bridge anchorages, etc., is certainly interesting and valuable, and while he is in no sense an alarmist, rather the contrary, with regard to the ultimate effect upon the bridge terminals of possible electrolytic action, the results which he has obtained should be an incentive to the proper authorities to take every possible precaution to avoid any possible detrimental effects.

The fact that the anchorages are securely and deeply embedded in cement should perhaps not be accepted too surely as a positive obstacle to electrolytic action, especially as high voltages may be placed upon the structure.

I have more than once pointed out that although the iron pipes of the New York subways are embedded in cement, and might therefore be considered as partially insulated, there has never been a time when the lead coverings of the cables therein have not furnished excellent "grounds." Dr J. A. Fleming, also, in a paper recently delivered before the British Association, on "The Electrolytic Corrosion of Water and Gas Pipes by the Return Currents of Electric Tramways," has expressed the opinion that the conduction through such materials as moist clay, cement, etc., must in great part at least be of an electrolytic character.

PROF. A. C. PECKHAM:—May I ask Mr. Knudson a question—if he knows whether this tie-rod was horizontal in the ground, or whether it stood edgewise. The question is based upon the idea that it may be difficult to account for one edge growing thin if it lay flatwise in the ground. But if it lay edgewise it might be natural to suppose that the lower edge would be the one that would grow thin.

MR. KNUDSON:—It lay edgewise in the ground, being an inch and a half wide. The lower part was the side that was cut the most. That is the part that was deepest in the ground.

PROF. PECKHAM:—It might be supposed that one part of the ground would be damper than another.

MR. KNUDSON:—They were all in just about the same condition.

MR. THATCHER T. P. LUQUER:—I would like to ask Mr. Knudson if he knows whether there is a return feeder for the Union railway on 135th street, or whether there is any method used for bonding the rails efficiently.

MR. KNUDSON:—I have seen no indication of any overhead return feeder at all. I presume, however, that the rails are bonded, but just how well they are bonded, of course I do not know, as they are covered up with earth.

[Information on this point raised by Mr. Maver and Prof. Peckham, received since reading the paper, shows that the presence of the ground wire is probably the cause of the tie-rods being eaten away at the middle.

The wire is usually placed on the ties midway between the rails, and consequently but a few inches below the rods. The escaping current in such case would pass from about the middle of the rods to the ground wire (the rods and rails being positive to earth and pipes,) and the excessive cutting at this portion of the rods would, without doubt, be accounted for. A. A. K.]

MR. JESSE M. SMITH:—I would like to ask Mr. Knudson for my own information, being a stranger in the city, how they propose to overcome the difficulty in the new lines on Sixth avenue and Broadway.

MR. KNUDSON:—Metallic circuits will be used. There are two conductors with the open conduit system, lying side by side. Of course there is little inducement for a current to be diverted from those conductors, and straying off on iron pipes, and taking some other course back to the station. It is confined to those conductors.

MR. SMITH:—Both insulated of course from the conduit?

MR. KNUDSON:—Both insulated.

MR. TOWNSEND WOLCOTT:—I would like to ask Mr. Knudson if he observed any great difference in the quality of the soil—if the chemical nature of the soil has anything much to do with electrolysis.

MR. KNUDSON:—No. I have not made any examination of the soils in the different parts of the city.

MR. WOLCOTT:—It is a little hard to understand just what becomes of metal chemically unless there is some acid of some kind present. I know it as a fact, (I have tried it myself), if you take two copper wires from a dynamo—(I think the dynamo I tried was about 300 volts) and stick them in a pail of ordinary well water, pretty nearly pure, the positive wire will be corroded away in a very short time. I believe it was a No. 4 wire I used. It was pointed as sharp as a needle in a few minutes by the electrolysis. But I do not know in what form the copper went into in solution. I did not make any chemical analysis or

anything of that kind. But there was the fact that nearly pure water will dissolve copper under the influence of current. Of course if there had been any acid in the water, it would have gone very much faster.

THE PRESIDENT:—Has the author any further remarks to make?

MR. KNUDSON:—I think not, with the exception perhaps regarding the last remark made by Mr. Wolcott, I would say that so far as my knowledge goes, the character of the soil is about the same in all of the cities; for instance that up town from where we got this sample, is shown to be sufficiently electrolytic to cause that much corrosion. I have no doubt that the same character of the soil exists in every part of New York city as well as in other cities. Some of the reports from Dayton, Ohio, which I did not quote, that are published in the pamphlet, show analysis of the character of the soil there, and it is evidently about the same, as it is here. In some cases the action on pipes has shown a covering something similar to graphite where electrolysis has acted upon it, and I have no doubt that the soil in most of the large cities of the country is of just about the same nature.

MR. WOLCOTT:—There are some parts of Manhattan Island where the soil is almost clean sand, if you go down a little depth, hardly any organic matter. I have no doubt that a soil, for example, that contained iron pyrites would corrode the wire very much faster on account of sulphuric acid being formed.

MR. JESSE M. SMITH:—I think that the soil of all cities certainly contains a great deal of animal matter, animal acids, due to the droppings of the horses and from nuisances of that kind in the street, making a soil which is admirably adapted for electrolytic action, particularly where there is water present, as there is especially nearly always in streets paved with stone.

I want to state a fact which is probably known to a great many of you, that in the city of Cincinnati they started out at the beginning with an overhead double trolley system of electrical transmission for railroads. They have persistently stood by that system, and I think it is probably the only city in the country where the double trolley is used to any great extent. My information is that they never have had any trouble whatever from electrolytic action in that city, although the electric lighting is largely done by underground conductors. But all of the trolley system, and it is a very large one, has overhead double conductors, and in that way there is no loss of current from the mains to the water pipes or gas pipes, and I understand that they find it a great economy to put the two conductors above ground where there is no leakage from one to the other.

[ADJOURNED.]

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

New York, Nov. 23rd, 1898.

The 129th meeting of the AMERICAN INTSITUTE OF ELECTRICAL ENGINEERS was held at 12 West 31st Street, this date, and was called to order by President Kennelly at 8:10 P. M.

THE SECRETARY: At the meeting of the Executive Committee this afternoon the following associate members were elected:

ABBE, CLEVELAND	Professor of Meteorology, The Weather Bureau, Washington, D. C., residence 2017 I St., N. W. Washington, D. C.	Jos. Wetzler, T. C. Martin. C. F. Chandler.
CASSIDY, JOHN	Superintendent Mutual Telephone Co., Honolulu, Hawaiian Islands, U. S. A.	T. C. Martin. Geo. H. Guy. Jos. Wetzler.
COSGROVE, JAMES FRANCIS	Head of Locomotive Engineering Dept. International Correspondence School, 631 Madison Ave., Scranton, Pa.	D. C. Jackson. J. Glen Wray. W. H. Donner.
FRY, DONALD HUME	Assistant to Robt. McF. Doble, 202 Sansome St., San Francisco, residence, Blue Lakes City, Cal.	Geo. P. Low. T. E. Theberath. F. P. Medina.
HAMILTON, JAMES	Patent Attorney and Expert in Patent Causes, 54 State St., Boston, Mass.	E. L. Nichols. H. J. Ryan. Fred'k. Bedell.
HOFFMANN, BERNHARD	New York Telephone Co., 15 Dey St., New York City.	Stephen D Field. U. N. Bethell. Herbert Laws Webb
JAMES, HARRY D.	Expert Draughtsman, Otis Bros. & Co., residence, 100 Buena Vista Av., Yonkers, N. Y.	W. M. Scott. C. W. Pike. Edwin R. Keller.
KNOX, S. L. G.	First Asst. Engineer, Crocker-Wheeler Electric Co., Ampere, residence, Montclair, N. J.	Gano S Dunn. F. V. Henshaw. F. M. Pedersen.
LEITCH, HOWARD WALLACE	Switchboard Regulator, The Edison Elect. Illuminating Co., residence, 378 Madison St., Brooklyn, N. Y.	Sam'l. Sheldon. Ralph W. Pope. J. W. Lieb, Jr.

LOHMANN, R. W.	Electrical Engineer, 1887 Madison Street, Oakland, Cal.	Geo. P. Low. Sidney Sprout. F. F. Barbour.
RENO, C. STOWE	Electrical Engineer, Triumph Electric Co., 620 Baymillen Street, Cincinnati, Ohio.	T. J. Creaghead. Thos. French, Jr. A. L. Searles.
SPERLING, R. H.	Assistant Engineer, British Columbia Electric Railway Co., Ltd. Victoria, B. C.	W. F. C. Hasson. F. F. Barbour. Wynn Meredith.
WILKINSON, JAMES	Station Manager, Chief Engineer, Consolidated Electric Light Co., 812 South 19th Street, Birmingham, Ala.	Geo. H. Harris. S. R. Gross. R. W. Pope.
ZAHN, A. WILFORD	Electrical Engineer and Supt., Manhattan Light, Heat and Power Co., Manhattan Building, St. Paul, Minn.	G. D. Shepardson. C. L. Pillsbury. A. A. Nimis.

Total 14.

The following Associate Members were transferred to full membership:

Approved by Board of Examiners, March 18th, 1898.

BROOKE-RIDLEY, A. E. Agent, Electrical Engineer, Siemens & Halske Electric Co., San Francisco, Cal.

Approved by Board of Examiners, October 20th, 1898.

FRANKLIN ROBERT ANSON. Secretary and Manager, Salem Light and Traction Co., Salem, Oregon.

SIDNEY HAND BROWNE Consulting Electrical Engineer, Baltimore, Md.

THE PRESIDENT: The paper for the evening, advance copies of which are in your hands, is on the Design of Alternating Current Transformers. It is somewhat lengthy, and it contains, as you will notice, a number of mathematical expressions. Such a paper is very difficult to have read aloud, but Prof. Franklin has kindly promised to present the salient features of this paper in such a manner that we will be able to follow it.

A paper presented at the 120th Meeting of the American Institute of Electrical Engineers, New York, November 23rd, 1898, President Kennelly in the Chair.

THE DESIGN OF TRANSFORMERS.

On the Best Proportions to Give to Transformer Cores, and the Most Advantageous Distribution of the Wasted Energy, Together with a Direct Method for the Design of Transformers, in Order to Obtain Maximum All-Day Efficiency.

BY FREDERICK W. CARTER.

I. *Introductory.*—In designing transformers it is usual to assume values for certain of the quantities which occur, and to determine the details of the transformer to suit the prescribed values of these quantities. Thus, the maximum induction to which the iron is subject, the proportions, or even the size of the cross-section of the magnetic circuit, the resistance of the coils and the maximum current density in them are often assumed. It is not, however, necessary in general that these quantities should have these prescribed values, and, in fact, in practice the effect of changes in the assumed quantities is tried, and the best transformer is really arrived at by a method of trial and error. Any such method is, however, tedious and imperfect in the case of a machine in which there are so many quantities that may be varied, and the present paper is intended principally to determine once for all the conditions of proportion and distribution of wasted power which lead to as little loss of energy as possible. The transformer may be made to satisfy these conditions by the method of trial and error, by successive approximation, or by any other method that may be thought suitable by the designer, but as the analysis used indicates a perfectly direct method of obtaining particulars of the transformer to satisfy the outside conditions and

to give minimum waste of energy with materials of given quality, the author has included a description of the method in the paper.

The assumptions on which the work is based are:

1. That the hysteresis loss is given by Steinmetz formula, that is, it varies as the volume of iron, as the frequency, and as some power of the maximum induction. (The 1.6th power is generally taken, but the results are worked out for a general index in Appendices B and C.)

2. That the eddy current loss varies as the volume of iron, as the square of the frequency, as the square of the thickness of the core plates, and as the square of the maximum induction.

3. That the ohmic loss varies as the resistance of the coils and as the square of the current in them.

These assumptions are probably sufficiently nearly correct for the work, especially over the range of induction which occurs.

All that we shall assume known at the outset concerning the transformer itself are:

1. The magnetic quality of the iron of the core.

2. The electric quality of the copper of the circuits.

3. The thickness of core plates and the amount of insulation between them: and often —

4. The maximum current density allowable in the coils.

The second of these is practically invariable, or so far as it varies, it chiefly depends on the temperature of working. The third is discussed in Appendix A. The fourth is usually assumed, because, as shown in section VII., some condition is necessary to fix the absolute size of the transformer.

The outside conditions supposed given are:

1. The primary and secondary voltages.

2. The frequency of alternation.

3. The normal full load.

4. The load time curve, as nearly as it can be predetermined.

5. The wave form, or the form factor, of the primary P. D. wave wherever possible.

The fourth of these is required because it is the "all-day" efficiency that is made a maximum, and the conditions of load should be known as nearly as they can be predetermined. When the fifth condition is known, the transformer can be designed to suit the wave form, which, although it has no effect on the proportions of the transformer, affects to some extent the absolute size and the number of convolutions of the wire.

The satisfaction of the condition of maximum efficiency, besides making the energy wasted as small as possible, will also give minimum heating for the amount of radiating surface, since the heat developed is a direct measure of the energy wasted. Again, the drop of secondary voltage as the load is increased depends principally on the ohmic resistance; a type, then, which has small ohmic losses should have this drop of voltage correspondingly small, so that making the waste of energy a minimum conduces to good regulation. The result will show that the shape of the core is not favorable to large magnetic leakage, so that the regulation on this account, too, should be good.

The transformers contemplated are of the closed-iron-circuit type and the load is supposed non-inductive.

II. *Sources of Waste Considered.*—The chief sources of loss of energy in a transformer are:

1. The ohmic loss in the primary and secondary coils.
2. The hysteresis loss in the core.
3. The eddy-current loss in the core.

Of these, the last is usually small compared with the two preceding it, and the calculations of the size of the various parts are best performed by neglecting it in the first instance, and then, if required, applying corrections to the results in order to take account of it. (See Appendix A).

III. *A Theorem on the Distribution of the Losses Between the Magnetic and Electric Circuits.*—The first proposition we shall prove is concerned with the distribution of loss of energy between the iron and copper circuits and may be stated thus:

"Given the iron circuit of a transformer of prescribed output, then, in order to make the wasted energy a minimum, the number of turns of wire in the coils must be such as to make the ohmic loss equal to 0.8 times the hysteresis loss, plus the eddy current loss."

This statement assumes that the index occurring in the Steinmetz formula is 1.6. Be it otherwise, a small obvious change must be made in the statement of the proposition; in fact, if it be ε then instead of 0.8 we must write $\varepsilon/2$ in our enunciation. The copper circuit, including its insulation, is supposed to take up all the space allotted to it, so that any increase in the number of turns in the coils necessitate a corresponding decrease in the cross-section of the wire. In other words, the *total* cross-section of each coil is known.

Let A be the total cross-section of copper in the primary.
l the mean length of a turn of the primary.
n the number of turns in the primary.
and I the primary current.

Then $\frac{A}{n}$ is the cross-section of the primary wire and

$$\rho \frac{l\,n}{\frac{A}{n}} \text{ or } \rho \frac{l}{A} n^2$$

is the resistance of the primary, where ρ is the specific resistance of copper. Thus the power wasted in the primary $=$

$$\rho \frac{l}{A} n^2 \; I^2 \propto n^2$$

Similarly the power wasted in the secondary varies as the square of the number of secondary turns, or as n^2, since the ratio of transformation is supposed given.

Thus the ohmic loss varies as n^2, and if we call it Ω, we may write $\Omega = \omega n^2$, where ω is independent of n.

The maximum number of lines of force through the magnetic circuit is

$$\frac{V_m\, 10^8}{2\pi f n}$$

at no load, where V_m is the maximum value of the potential difference between the primary terminals, and f is the frequency. The number is very slightly less at full load, but we are only concerned with the fact that it varies inversely as n at all safe loads. Hence the induction, which is this divided by the area of cross-section of the magnetic circuit, varies inversely as n. Hence the hysteresis loss, which varies as the induction to the power of 1.6, varies as $n^{-1.6}$.

Thus if H be the hysteresis loss, we may write $H = h\, n^{-1.6}$, where h is independent of n.

Again, the eddy current loss varies as the square of the induction, and therefore as n^{-2}.

Thus if E be the eddy current loss, we may write $E = e\, n^{-2}$, where e is independent of n. Hence if W be the total loss,

$$\begin{aligned} W &= \Omega + H + E \\ &= \omega n^2 + h\, n^{-1.6} + e\, n^{-2} \end{aligned}$$

thus

$$\frac{\delta W}{\delta n} = 2\,\omega\,n - 1.6\,h\,n^{-2.6} - 2\,e\,n^{-3}.$$

This is zero when W is a minimum,

$$\therefore \frac{2}{n}(\omega\,n^2 - 0.8\,h\,n^{-1.6} - e\,n^{-2}) = 0$$

or $\mathcal{Q} = 0.8\,H + E$, which proves the proposition.

The fact that this solution corresponds to a minimum value of W, and not to a maximum, can be seen from the nature of the case, or from the fact that

$$\frac{\delta^2 W}{\delta n^2} = 2\,\omega + 1.6 \times 2.6\,h\,n^{-3.6} + 2 \times 3\,h\,n^{-4}$$

which is necessarily positive.

IV. *Extension of the theorem to the case of a variable load.*— The above conclusion is true whether the load is constant or variable, provided that, in the latter case, by the various losses we understand the time average of those losses. For if W be the time average of the total loss, using the same notation as in the last section, we shall have

$$W = \frac{1}{T}\int_0^T \omega\,n^2\,dt + \frac{1}{T}\int_0^T h\,n^{-1.6}\,dt + \frac{1}{T}\int_0^T e\,n^{-3}\,dt$$

where ω, h, and e, are now functions of t.

Thus

$$\frac{\delta W}{\delta n} = \frac{2}{T}\int_0^T \omega\,n\,dt - \frac{1.6}{T}\int_0^T h\,n^{-1.6}\,dt - \frac{2}{T}\int_0^T e\,n^{-3}\,dt$$

Hence, if

$$\frac{\delta \omega}{\delta n} = 0$$

$$\frac{1}{T}\int_0^T \omega\,n^2\,dt = \frac{0.8}{T}\int_0^T h\,n^{-1.6}\,dt + \frac{1}{T}\int_0^T e\,n^{-3}\,dt$$

or

$$\mathcal{Q} = 0.8\,H + E$$

where $\mathcal{Q}$, H, and E, are now the time averages of the ohmic loss, the hysteresis loss, and the eddy-current loss, respectively.

V. *Limits of the theorem.*—It is to be understood that the theorems of the last two sections refer to a transformer whose core is given. In a transformer where some other condition is prescribed, we might find the relation between ohmic and iron loss, required to give minimum total loss, to be quite other than that found above; but having so determined the transformer, we can, with the same core, always find a copper circuit of the same weight, which will give less total loss, but it will only be by violating the prescribed condition. For instance, if the ohmic loss is prescribed, we shall have the total loss a minimum when the iron loss is as small as possible. Thus, a large core, and a large number of turns of thick wire, will give less loss than a core designed to make the ohmic loss equal to 0.8 times the hysteresis loss together with the eddy-current loss; but, given that larger core, we can, with the same weight of copper make a more efficient transformer by satisfying this relation, though it will, by diminishing the ohmic loss, violate the prescribed condition.

In fact, if the relation proved above is not satisfied, we can, without altering the core, and without altering the weight of the copper, (and so, practically, without altering the cost,) improve the transformer, as regards efficiency, by satisfying this relation. So that, hereafter, in designing transformers, we shall usually assume that the above relation is to be satisfied as well as the prescribed conditions.

VI. *Direct application of the theorem.*—The results of sections 3 and 4 enable us immediately to find the number of turns of wire in the primary and secondary coils, which will make the efficiency a maximum, when the core is given. For if, as before Ω, H, and E, be the average values of the ohmic loss, the hysteresis loss, and the eddy-current loss respectively, and we write

$$\Omega = \omega n^2, \qquad H = h n^{-1.6}, \qquad E = e n^{-2},$$

where ω, h, e, are independent of n, then, to a first approximation

$$\Omega = 0.8\, H,$$

or
$$\omega n^2 = 0.8\, h n^{-1.6},$$

or
$$n^{3.6} = 0.8 \frac{h}{\omega},$$

and h, ω, are known in terms of the size of the core, and the quality of the materials.

The correction to be applied to this to take account of eddy-current loss is easily made.

For suppose n given by the equation above, and $n + \delta n$ to satisfy

$$Q = 0.8\ H + E, \text{ or that}$$

$$\omega\ (n + \delta n)^2 = 0.8\ h\ (n + \delta n)^{-1.6} + e\ n^{-2}.$$

Thus

$$2\ n^2\ \omega\ \frac{\delta n}{n} = -\ 0.8 \times 1.6\ \ h\ n^{-1.6}\ \frac{\delta n}{n} + \ e\ n^{-2},$$

neglecting squares of small quantities.

This may be written

$$2\ n^2\ \omega\ \frac{\delta n}{n} + 1.6\ n^2\ \omega\ \frac{\delta n}{n} = e\ n^{-2},$$

or

$$3.6\ n^3\ \omega\ \frac{\delta n}{n} = e\ n^{-2},$$

or

$$\delta n = \frac{e\ n^{-2}}{3.6\ \omega\ n^2} n.$$

The calculations required to find ω, h, and e, will be given hereafter. (§ § XVII. and XVIII.)

VII. *Necessity of a condition to determine the size of the transformer.*—If the *proportions only* of the core are given, and not the actual size, then, as stated in the first section, some condition will be necessary to limit the actual size, as there is no transformer whose waste of energy is an absolute minimum, when only the quality of the materials and the external conditions are given. This can be seen as follows:—

Suppose the linear dimensions of the transformer are increased x times in all parts, except the *thickness* of the core plates, which is to remain constant.

Since the output and voltage are given, the currents are practically given, hence the ohmic loss varies as the resistances that is as $\frac{\text{length of wire}}{\text{area of cross-section}}$ or as $\frac{1}{\text{linear dimensions}}$ or as $\frac{1}{x}$.

Again, since the voltage and frequency are given, and the number of turns of wire remains constant, the total flux of

magnetic lines remains constant, hence the induction, or flux per unit area varies as $\frac{1}{x^2}$.

Now the hysteresis loss varies as the volume of iron and the 1.6th power of the induction, that is as $x^3 (x^{-2})^{1.6}$, or as

$$x^3 \times x^{-3.2}, \text{ or as } \frac{1}{x^{0.2}}.$$

And the eddy current loss varies as the volume of iron and the square of the induction, that is as $x^3 (x^{-2})^2$, or as

$$x^3 \times x^{-4}, \text{ or as } \frac{1}{x}.$$

Thus this increase of the transformer diminishes the ohmic and eddy current losses x times, and the hysteresis loss $x^{0.2}$ times; so that all the losses are decreased by the increase of the size.

Of course, if the index of the power of the induction in Steinmetz's formula were less than 1.5, the hysteresis loss would increase slightly with increase of size, but as exact proportionality need not be aimed at in the increase, there is little doubt that, by slightly increasing the number of turns of wire on the coils, we could always make all the losses decrease, by increasing the size of the transformer.

Thus, we see that some condition is necessary to determine the absolute size of the transformer. Practically this is limited, on the one hand by cost, and on the other, by considerations of safety, performance, and economy. The ideal way of proceeding with the design would be to allow these limits to determine the size. They, however, are not definite, and are incapable of being accurately expressed, in any simple manner, in terms of the linear dimensions. In the last section the size was determined by the size of the core. In order to investigate what core will best satisfy our conditions, we shall assume that the maximum current density in the copper is given. This is a usual assumption to make in designing transformers, and the current densities practically used are well known. The ohmic loss per unit volume of copper is determined by the current density, which is, accordingly, intimately connected with the heating, and which will not vary very much between different transformers of the same standard of excellence.

VIII. *Principles of the design of a transformer whose proportions are given.*—We will now discuss the question of the actual size of a transformer whose proportions are given, which

will satisfy the relation proved in sections 3 and 4, and be such that the maximum current density in the copper is given.

Since the power and voltage are given, the maximum current in each coil is practically known; so that, to be given the maximum current density, is virtually to be given the size of the wire used for the coils.

Let a, b, c,—be proportional to linear dimensions of the transformer.

Let $x\,a$, $x\,b$, $x\,c$,—be the corresponding actual linear dimensions. Since the cross-section of the wire is given, the number of turns, (n), varies as the area of the winding space, or $n \propto x^2$.

Now the induction varies inversely as n and inversely as the area of cross-section of the magnetic circuit, and therefore, inversely as x^4.

Hence the hysteresis loss, which varies as the volume of iron, and as the 1.6th power of the induction, varies as $x^3\ (x^{-4})^{1.6}$, or as $x^{-3.4}$.

Thus if we call it H, we may write,

$$H = h_1\, x^{-3.4}, \text{ where } h_1 \text{ is independent of } x.$$

Similarly, the eddy-current loss varies as the volume of iron, and the square of the induction, or as $x^3\,(x^{-4})^2$, or as x^{-5}.

Thus if we call it E, we may write

$$E = e_1\, x^{-5}, \text{ where } e_1 \text{ is independent of } x.$$

The ohmic loss, since the maximum current density is given, varies as the volume of copper, or as x^3.

Thus if we call it Ω, we may write

$$\Omega = \omega_1\, x^3 \text{ where } \omega_1 \text{ is independent of } x.$$

Thus x is given by the equation

$$\omega_1\, x^3 = 0.8\, h_1\, x^{-3.4} + e_1\, x^{-5}.$$

To solve this, we may first neglect E, and so get

$$x^{6.4} = 0.8\, \frac{h_1}{\omega_1}$$

which gives a first approximation to x.

The correction to be applied to this to take account of the eddy-current loss may be found thus:

Calling the correction to be applied to x, δx, we shall have δx given by

$$\omega_1\,(x + \delta x)^3 = 0.8\, h_1\,(x + \delta x)^{-3.4} + e_1\,(x + \delta x)^{-5},$$

where x is given by

$$\omega_1\, x^3 = 0.8\, h_1\, x^{-3.4};$$

thus

$$3\ \omega_1 x^3 \frac{\delta x}{x} = -3.4 \times 0.8\ h_1 x^{-3.4} \frac{\delta x}{x} + e_1 x^{-5},$$

or

$$6.4\ \omega_1 x^3 \frac{\delta x}{x} = e_1 x^{-5},$$

or

$$\delta x = \frac{e_1 x^{-5}}{6.4\ \omega_1 x^3} \cdot x$$

The calculations required to find $\omega_1 h_1$ and e_1 will be given hereafter. (§§ XVII and XVIII.)

IX. *Discussion of the same design, when the conditions of* §§ III *and* IV *are not assumed.*—It must be observed that the problem of the last section is quite different from that of finding a core which will give the greatest efficiency with the given cross-section of wire, and, just as in § V we found that, for a minimum loss of energy, the core would have to be infinite, when the ohmic loss is prescribed, so here we may show that the core which gives least wasted energy when the cross-section of the wire is prescribed, is larger than is required to give the best distribution of losses between iron and copper.

For, using the same notation as in the last section, if W be the total loss,

$$W = \Omega + H + E,$$
$$= \omega_1 x^3 + h_1 x^{-3.4} + e_1 x^{-5}$$
$$\frac{\delta W}{\delta x} = 3\ \omega_1 x^3 - 3.4\ h_1 x^{-4.4} - 5\ e_1 x^{-6}$$

Hence if W is a minimum

$$\omega_1 x^3 = \frac{3.4}{3} h_1 x^{-3.4} + \frac{5}{3} e_1 x^{-5},$$

or

$$\Omega = 1.13\ H + 1.67\ E.$$

Thus

$$\Omega > 0.8\ H + E.$$

Hence, if we increased the cross-section of the wire and diminished the number of turns, we should diminish the ohmic loss, and increase the iron loss, but we should diminish the total loss. The increase of the cross-section of the wire is not a detrimental variation, so that, as a rule, it will be advantageous to assume that the given cross-section of the wire is that which makes the energy wasted as small as possible *with the core found;* in

other words, the method of design given in the last section is the appropriate one to apply to the problem.

X. *The shell transformer, which gives least waste of energy, considered.*—Thus we see how, when we are given the proportions of the core, we can find the size which will give least wasted energy, subject to the condition that the maximum current density is prescribed. We will now investigate the subject of the best proportions to give to the core, in order that the efficiency may be a maximum when the maximum current density is prescribed as before. We shall also assume that the copper circuit is that which gives greatest efficiency with the core found, in other words, we shall assume the condition of sections 3 and 4.

It will be necessary to fix upon the general type of the transformer, and I shall deal first with one of the ordinary shell type, in which the dimensions a, b, c, l, are as indicated in the accompanying diagram. (Fig. 1.)

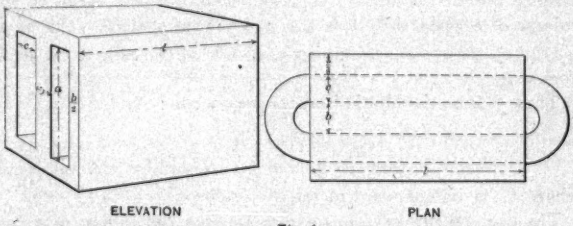

Fig. 1

Since the maximum current in the coils is known, as also the maximum current density, the cross-section of the wires is known. Hence the number of turns in a coil varies as its cross-sectional area, that is as the winding space. Hence, if n be the number of primary turns —

$$n \text{ varies as } a\ c.$$

Again, the magnetic induction varies inversely as the cross-section of the magnetic circuit, and inversely as n, that is, inversely $ac.\ bl.$

The volume of iron varies as the measured volume of the core, that is as $2\,b\,l\,(a+b+c)$, or as $b\,l\,(a+b+c)$.

The hysteresis loss varies as the volume of iron and as the 1.6th power of the induction, that is, as $b\,l\,(a+b+c)\;(a\,c\,b\,l)^{-1.6}$.

Thus if H be the hysteresis loss, we may write

$$H = H_1\,(a+b+c)\;(b\,l)^{-0.6}\,(a\,c)^{-1.6},$$

where H_1 is independent of the dimensions.

The eddy-current loss varies as the volume of iron and as the square of the induction, that is as $b\,l\,(a+b+c)\;(a\,c\,b\,l)^{-2}$.

Thus if E be the eddy-current loss, we may write :—

$$E = E_1\,(a+b+c)\;(b\,l)^{-1}\,(a\,c)^{-2}$$

where E_1 is independent of the dimensions.

Since the current density is given, the ohmic loss varies as the volume of copper, that is as the volume of the coils, that is as $a\,c\,(2\,l+\pi\,\overline{b+c})$, (assuming the ends of the coils to be semicircular.)

Thus if Ω be the ohmic loss, we may write :—

$$\Omega = \Omega_1\,a\,c\,(2\,l+\pi\,\overline{b+c}),$$

where Ω_1 is independent of the dimensions.

The problem we have to solve, is to find the values of a, b, c and l, which make $\Omega + H + E$ a minimum, subject to the condition, $\Omega = 0.8\,H + E$, or as we may state it, we have to make the total loss,—which we will call W, and write $W = \Omega + H + E + K\,(\Omega - 0.8\,H - E)$, a minimum where K is a constant so chosen that $\frac{\delta\,W}{\delta\,l} = 0$ identically for the values of a, b, c, l, which give maximum efficiency, and a, b, c, are now independent variables.

Hence

$$\left.\begin{aligned}
&\frac{\delta W}{\delta l} = (1+K)\,\mathcal{Q}\,\frac{2}{2l+\pi b+c} - (1-0.8\,K)\,H\,\frac{0.6}{l} \\
&\qquad\qquad - (1-K)\,E\,\frac{1}{l} = 0 \\
&\frac{\delta W}{\delta a} = (1+K)\,\mathcal{Q}\,\frac{1}{a} + (1-0.8\,K)\,H\left[\frac{1}{a+b+c} - \frac{1.6}{a}\right] \\
&\qquad + (1-K)\,E\left[\frac{1}{a+b+c} - \frac{2}{a}\right] = 0 \\
&\frac{\delta W}{\delta b} = (1+K)\,\mathcal{Q}\,\frac{\pi}{2l+\pi b+c} + (1-0.8\,K)\,H \\
&\left[\frac{1}{a+b+c} - \frac{0.6}{b}\right] + (1-K)\,E\left[\frac{1}{a+b+c} - \frac{1}{b}\right] = 0 \\
&\frac{\delta W}{\delta c} = (1+K)\,\mathcal{Q}\left[\frac{\pi}{2l+\pi b+c} + \frac{1}{c}\right] + (1-0.8\,K)\,H \\
&\left[\frac{1}{a+b+c} - \frac{1.6}{c}\right] + (1-K)\,E\left[\frac{1}{a+b+c} - \frac{2}{c}\right] = 0
\end{aligned}\right\} \text{I}$$

In order to solve these, we will, as usual, first find an approximation to the solution by neglecting the eddy currents.

Thus we get

$$\frac{(1+K)\,\mathcal{Q}}{(1-0.8\,K)\,H} = \frac{0.3\,[2l+\pi b+c]}{l} = \frac{1.6\,b+c+0.6\,a}{a+b+c}$$

$$= \frac{[2l+\pi b+c]\,[0.6\,a+c-0.4\,b]}{\pi b\,(a+b+c)} = \frac{[1.6\,a+b+0.6\,c]\,[2l+\pi b+c]}{[a+b+c]\,[2l+\pi b+2c]}$$

and $\mathcal{Q} = 0.8\,H$.

These have the unique solution

$$\frac{l}{9\pi} = \frac{a}{14} = \frac{b}{9} = \frac{c}{7}$$

and $$K = \frac{1}{5.12}$$ (See Appendix B).

Thus the proportions of l, a, b, and c, are found, and their absolute values are given by the equation

$$\mathcal{Q} = 0.8\,H.$$

Had the solution not been restricted by the condition $\mathcal{Q} = 0.8\ H$, we should have found the same proportions for l, a, b, and c, but their absolute values would have been given by $\mathcal{Q} = 1.13\ H$, which latter fact also follows from § IX since the proportions are the same as when the equation $\mathcal{Q} = 0.8\ H$ is satisfied.

These results follow immediately by putting $K = 0$ in the equation above.

As is shown in § IX, however, we shall do right to assume that the conditions expressed by the equation $\mathcal{Q} = 0.8\ H$, are satisfied.

Thus the absolute valne of the dimensions is given by this relation, and, expressing them all in terms of c, we get

$$c^{8.4} = 0.0138\ \frac{H_1}{\mathcal{Q}_1}$$

So that we have found the first approximation to the required values of l, a, b, and c. We can now find the corrections to be applied to the values so found. To do this, suppose the corrections to be applied to l, a, b, and c, to be $\lambda\ l$, $\alpha\ a$, $\beta\ b$, and $\gamma\ c$, respectively, so that the correct values of the magnitudes in question are $l\ (1 + \lambda)$, $a\ (1 + \alpha)$, $b\ (1 + \beta)$, and $c\ (1 + \gamma)$, respectively; then, if $\mathcal{Q}'$, H', and E', are the first approximations to the ohmic, the hysteretic and the eddy losses respectively, the true value of these losses to the second approximation will be

$$\mathcal{Q}'\left(1 + \frac{34\ \alpha + 9\ \beta + 41\ \gamma + 18\ \lambda}{34}\right),$$

$$H'\left(1 - \frac{34\ \alpha + 9\ \beta + 41\ \gamma + 18\ \lambda}{30}\right),$$

and E respectively, and equations I. of this section reduce to

$$6.12\ \alpha + 0.86\ \beta + 6.79\ \gamma + 4.59\ \lambda = 3.65\ \frac{E'}{\mathcal{Q}'} - \delta\ K,$$

$$6.75\ \alpha + 1.27\ \beta + 7.10\ \gamma + 3.24\ \lambda = 2.96\ \frac{E'}{\mathcal{Q}'} - \delta\ K,$$

$$4.78\ \alpha + 5.74\ \beta + 6.12\ \gamma + 1.72\ \lambda = 5.11\ \frac{E'}{\mathcal{Q}'} - \delta\ K,$$

$$5.89\ \alpha + 1.34\ \beta + 8.14\ \gamma + 2.98\ \lambda = 2.83\ \frac{E'}{\mathcal{Q}'} - \delta\ K.$$

Whilst the equation

$$\mathcal{Q} = 0.8\, H + E$$

reduces to

$$34\, \alpha + 9\, \beta + 41\, \gamma + 18\, \lambda = 15.9\, \frac{E'}{\mathcal{Q}'}.$$

The solution of these equations is

$$\alpha = \gamma = -\, 0.036\, \frac{E'}{\mathcal{Q}'},$$

$$\beta = \lambda = 0.69\, \frac{E'}{\mathcal{Q}'},$$

$$\delta K = 0.36\, \frac{E'}{\mathcal{Q}'}.$$

(See Appendix C).

To sum up then, the first approximation to the values of l, a, b, c, is given by

$$\frac{l}{9\pi} = \frac{a}{14} = \frac{b}{9} = \frac{c}{7}$$

and

$$c^{6.4} = 0.0138\, \frac{H_1}{\mathcal{Q}_1}$$

whilst the corrected values of these magnitudes are got by increasing l and b each by $0.69\, \frac{E'}{\mathcal{Q}'}$ of itself, and diminishing a and c each by $0.036\, \frac{E'}{\mathcal{Q}'}$ of itself.

The calculations proposed to find H_1, $\mathcal{Q}_1$, E' and $\mathcal{Q}'$ will be given hereafter. (§ XVII.)

XI. *The core transformer, which wastes least energy, considered.*—We will next consider the size and proportions to give to a transformer of the ordinary core type, in order that the total energy wasted may be a minimum, the maximum current density being, as before, prescribed. We shall again suppose the conditions of § § III and IV to be satisfied.

We will suppose that the shape of the core is as in Fig. 2, that the coils are wrapped on both vertical limbs, and that the wires are straight where they pass the edges of the plates, and semicircular across the faces. Let the dimensions a, b, c, and l, be as shown in Fig. 2.

Exactly as in the last section we shall have

1. Number of turns of wire varies as $a\,c$.
2. The induction varies as $(a\,c.\ b\,l)^{-1}$.

But the volume of iron is now proportional to

$$b\,l\,(a+c+2\,b),$$

and the volume of copper is proportional to

$$a\,c\,(2\,l+\pi\,\overline{b+\frac{c}{2}})$$

Thus the hysteresis loss, which we will call H, is

$$H = H_1\,b\,l\,(a+c+2\,b)\,(a\,c\,b\,l)^{-1\cdot6},$$

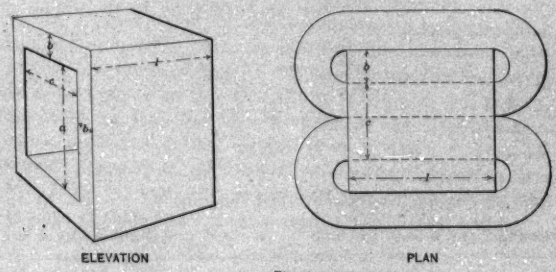

Fig. 2

where H_1 is independent of the dimensions.

The eddy-current loss, which we will call E, is

$$E = E_1\,b\,l\,(a+c+2\,b)\,(a\,c\,b\,l)^{-2},$$

where E_1 is independent of the dimensions.

The ohmic loss, which we will call Ω, is

$$\Omega = \Omega_1\,a\,c\,\Big(\ 2\,l+\pi\,\overline{b+\frac{c}{2}}\Big),$$

where Ω_1 is independent of the dimensions.

If we write $a = 2\,a'$, $c = 2\,c'$ we shall get,

$$H = H_1\,.2\times4^{-1\cdot6}\,b\,l\,(a'+b+c')\,(a'\,c'\,b\,l)^{-1\cdot6},$$

$$E = E_1 .2 \times 4^{-3}\, b\, l\, (a' + b + c')\ (a'\ c'\ b\, l)^{-3},$$

$$\mathcal{Q} = \mathcal{Q}_1 .4\ a'\ c'\ [2\ l + \pi\ \overline{b + c'}].$$

Thus we have to choose $a'\ b\ c'\ l$ so that

$\mathcal{Q} + H + E$ may be a minimum, when $\mathcal{Q} = 0.8\ H + E$.

This is exactly the same problem as that of the last section, and the first approximation to the solution will be

$$\frac{l}{9\,\pi} = \frac{a'}{14} = \frac{b}{9} = \frac{c'}{7},$$

where

$$c'^{6.4} = 0.0138 \times \frac{2 \times 4^{-1.6}}{4}\ \frac{H_1}{\mathcal{Q}_1},$$

or

$$c^{6.4} = 0.0633\ \frac{H_1}{\mathcal{Q}_1}.$$

Thus we obtain the first approximation to the dimensions required. The correction for the eddy currents is the same as for the shell transformer.

Thus, summing up the results for a core transformer, the first approximation to the values of l, a, b, and c, is given by

$$\frac{l}{9\pi} = \frac{a}{28} = \frac{b}{9} = \frac{c}{14}, \qquad \text{and } c^{6.4} = 0.0633\ \frac{H_1}{\mathcal{Q}_1},$$

whilst the corrected values of these magnitudes are obtained by increasing l and b each by $0.69\ \frac{E'}{\mathcal{Q}'}$, of itself, and decreasing a and c each by $0.036\ \frac{E'}{\mathcal{Q}'}$, of itself.

The calculations for finding H_1, $\mathcal{Q}_1$, E', $\mathcal{Q}'$, will be given hereafter. (§ XVIII.)

XII. *Remarks on the two preceding sections.*—Thus we have found the actual dimensions of the cores of transformers, both of the shell and core type, which will make the loss of energy a minimum, when the maximum current density is prescribed and the coils are such that no change in them would make the transformer more efficient. These dimensions are given in terms of quantities which are known when we know the quality of the materials, and the external and prescribed conditions.

We shall next proceed to show in detail how to express the quantities involved, in order to directly calculate the particulars of the transformer. In any particular case, the calculations can be made, by means of a slide rule, with very little trouble.

If several not very different transformers have to be designed, an approximate value of the eddy current correction might be found which would be sufficiently near to apply to all—this especially when the particulars of the iron are not accurately known. The increase of b and l due to the eddy current losses will rarely be greater than 0.2, and the decrease of a and c is of the order 0.01, which might in many cases be neglected. It may be mentioned that, since the quality of transformer iron deteriorates a little at first, the constants should be taken for aged iron.

The solution given corresponds to an absolutely minimum waste of energy, as will be shown in Appendix D.

XIII. *Determination of the average losses in terms of the maximum.*—We will first show how we may find, approximately, the average values of the various losses when we are given the pre estimated relation between load and time. We may regard the load as measured by the inverse of the resistance in the secondary circuit. If we call this resistance r, the primary voltage V, the resistances of the primary and secondary transformer coils R and R' respectively, and their number of turns n and n', we may take it as sufficiently near for our purpose to write for the primary current (I)

$$I = \frac{n'^2}{n^2} \frac{V}{r}$$

and for the secondary current (I')

$$I' = \frac{n'}{n} \frac{V}{r}$$

Hence the ohmic loss (Ω) at this load is

$$\Omega = R I^2 + R' I'^2,$$

$$= R \frac{n'^4}{n^4} \frac{V^2}{r^2} + R' \frac{n'^2}{n^2} \frac{V^2}{r^2},$$

$$= \left[R \left(\frac{n'}{n}\right)^2 + R' \right] \left(\frac{n'}{n}\right)^2 \frac{V^2}{r^2}.$$

If Q_0 be the ohmic loss at full load, and r_0 the corresponding resistance

$$Q_0 = \left[R \left(\frac{n'}{n}\right)^2 + R' \right] \left(\frac{n'}{n}\right)^2 \frac{V^2}{r_0^2},$$

$$\therefore Q = Q_0 \frac{r_0^2}{r^2}.$$

The average ohmic loss $\overline{Q}$ is given by

$$\overline{Q} = Q_0 r_0^2 \frac{1}{T} \int_0^T \frac{dt}{r^2} = Q_0 \frac{r_0^2}{\overline{r^2}}$$

where $\frac{1}{\overline{r^2}}$ is the mean value of $\frac{1}{r^2}$

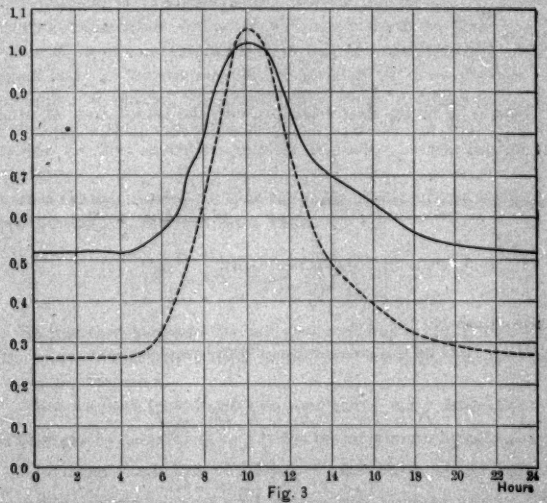

Fig. 3

[The full line curve is between load and time, the ordinate being the fraction of full load. The dotted curve is between square of load and time, the ordinate being the square of that of full line curve.

For this curve

$$k = 0.45.]$$

Hence, if we are given the curve Fig. 3, between the load, $\left(\frac{1}{r}\right)$ and the time, and from it deduce the curve between the square of the load and the time, the ratio of the mean to the maximum ordinate of this latter curve, is the ratio of the mean to the maximum ohmic loss.

The ratio so found, we shall call k, and write

$$Q = k\, Q_w.$$

This approximation will probably be as near as can be relied on, as the load curve can in general hardly be pre-estimated with very great accuracy.

The average value of the hysteresis loss will perhaps be very near to the maximum value, that is, within perhaps two per cent. We can, however, find this average as follows:—

The induction, as far as it depends on the load, varies inversely as $1 + \frac{n'^2}{n^2}\,\frac{R}{r}$ nearly; hence the hysteresis loss varies as

$$\left(1 + \frac{n'^2}{n^2}\,\frac{R}{r}\right)^{-1.6}.$$

Thus if H be the hysteresis loss with the load $\frac{1}{r}$, and H_0 that at no load

$$H = H_0 \left(1 + \frac{n'^2}{n^2}\,\frac{R}{r}\right)^{-1.6},$$

$$= H_0 \left(1 - 1.6\,\frac{n'^2}{n^2}\,\frac{R}{r}\right).$$

approximately.

Thus, if $\overline{H}$ be the mean value of the hysteresis loss

$$\overline{H} = H_0 - 1.6\,\frac{n'^2}{n^2}\,R\,H_0\,\frac{1}{T}\int_0^T \frac{d\,t}{r},$$

$$= H_0 \left[1 - 1.6\,\frac{n'^2}{n^2}\,\frac{R}{\overline{r}}\right],$$

where $\frac{1}{\overline{r}}$ is the mean ordinate of the load-time curve.

Thus the maximum hysteresis loss H_0 must be diminished by

$$1.6 \frac{n'^2}{n^2} R H_0$$

times the mean load, $\frac{1}{r}$, to obtain the mean hysteresis loss.

Thus if we write

$$1 - 1.6 \frac{n'^2}{n^2} \frac{R}{r} = j, —$$

$$\overline{H} = j H_0.$$

We may notice that

$$\frac{n'^2}{n^2} R = R',$$

nearly, if the maximum current density is to the same in primary and secondary. It is also to be observed that R and R' are not known at the outset, but as R' will hardly be more than $\frac{1}{100}$ of $\bar{r}$, the effect of a considerable error in R' will have very little effect on j; so that we may either estimate R' from former experience, or, design the transformer roughly, with $j = 1$, and then correct it for the small change in j, if such a degree of accuracy as the correction implies is really obtainable in the quantities.

In a similar manner, if $\overline{E}$ be the mean value of the eddy-current loss and E_0 the maximum value.

$$\overline{E} = E_0 \left[1 - 2 \frac{n'^2}{n^2} \frac{R}{r} \right].$$

So that the change in the eddy-current loss is small, and since the eddy-current loss is itself small, we may take the maximum value for the mean.

Thus we have introduced two constants j and k, such that

The mean hysteresis loss $= j$ times the maximum hysteresis loss.

The mean ohmic loss $= k$ times the maximum ohmic loss.

and have shown how to obtain them from the load curve. We may note that k may be anything less than unity, whilst j is very nearly unity but somewhat less.

We shall preserve this notation for these constants throughout.

XIV. *Introduction of a constant determined by the insulation of the coils.*—We will next define a constant which will be required in writing down the general expression for the ohmic loss, and which depends upon the proportion of insulation between the coils. This is the ratio of the total cross-section of copper in the primary or secondary circuit, to the cross-section of both coils, that is, of the winding pocket. When the maximum current density is given, the size of wire for the primary and secondary circuits is known, so that the proportion of the coils taken up by insulation can be predetermined with considerable accuracy.

Let σ, σ' be the cross-sections of the wires of the primary and secondary respectively, $s\sigma$, $s'\sigma'$, the average cross-section of wire and insulation corresponding (wherein s, s', not only include the immediate covering of the wire, but the insulation and ventilation spaces between layers and coils as well; in fact $s\sigma$ is the quotient of the total cross-section of the primary coil, by the number of primary turns).

Let n, n', be the number of primary and secondary turns respectively.

Then

$n s \sigma + n' s' \sigma' =$ the total area of the winding pocket. $= A$ let us say. (A is the "$a.c$" of §§ x and xi.)

Now if the maximum current density is to be the same in both primary and secondary $n\sigma = n'\sigma'$, and if we write each of the quantities equal to $p A$, we shall have

$$p A (s + s') = A.$$

$$\therefore p = \frac{1}{s + s'}.$$

s and s' are both greater than 1, so that p is less than a half.

This, (p), is the constant we shall require.

XV. *Constants depending on the insulation of the core plates.*—Two constants will also be required depending on the insulation of the core plates; these can be determined immediately when we know the thickness of the plates, and that of the insulation.

We will call them q and q', and they are defined thus:

q is the ratio of the *volume of iron* in the core, to the *measured volume* of the core.

q' is the ratio of the area occupied by iron in the cross-section of the magnetic circuit, to the total cross-section.

As a rule $q = q'$.

The five constants p, q, q', j, k, are all, then, pre-determinable with considerable accuracy, and these are all that we shall require, besides those depending on the quality of the materials, viz. ρ—the specific resistance of copper, η—the hysteretic constant in Steinmetz's formula, and δ—a corresponding constant in the eddy-current formula.

XVI. *Detailed expressions for the losses in a shell transformer.*—We will now obtain general expressions for the various losses, in terms of the dimensions of the core of the transformer and the number of turns of wire on the coils, which number we shall express in terms of the dimensions when the maximum current density is given.

We will first consider a transformer of the shell type; and will suppose the dimensions a, b, c, l, to be given by Fig. 1.

If the maximum currents in the coils are I and I' respectively their number of turns n and n' and their resistances R and R', then the maximum ohmic loss of power is

$$\mathfrak{Q}_0 = R\,I^2 + R'\,I'^2.$$

Now, the mean length of a turn of wire is $2\,l + \pi\,(b + c)$, and the cross-section of the primary wire is $\frac{p\,a\,c}{n}$, and of the secondary wire $\frac{p\,a\,c}{n'}$.

Thus

$$R = \rho\,\frac{(2\,l + \pi\,\overline{b + c}).n}{\frac{p\,a\,c}{n}} = \rho\,\frac{2\,l + \pi\,\overline{b + c}}{p\,a\,c}\,n^2,$$

and

$$R' = \rho\,\frac{2\,l + \pi\,\overline{b + c}}{p\,a\,c}\,n'^2.$$

Thus

$$\mathfrak{Q}_0 = \rho\,\frac{2\,l + \pi\,\overline{b + c}}{p\,a\,c}\,(n^2\,I^2 + n'^2\,I'^2),$$

$$= 2\,\rho\,\frac{2\,l + \pi\,\overline{b + c}}{p\,a\,c}\,n^2\,I^2,$$

approximately.

The mean ohmic loss is $\overline{\mathcal{Q}} = k\,\mathcal{Q}_0$, or

$$\overline{\mathcal{Q}} = 2\,k\,\rho\,\frac{2\,l + \pi\,\overline{b+c}}{p\,a\,c}\,n^2\,I^2. \qquad (1)$$

If the maximum current density is given,

$$\frac{n\,I}{p\,a\,c}$$

is known.

Thus if we write

$$\frac{n\,I}{p\,a\,c} = \delta,$$

the current density, in amperes per sq. cm., and

$$\frac{p\,a\,c}{n} = \sigma,$$

the cross-section of the primary wire,

$$\overline{\mathcal{Q}} = 2\,k\,\rho\,p\,a\,c\,(2\,l + \pi\,\overline{b+c})\,\delta^2 \qquad (2)$$

Again if V and V', be the primary and secondary voltages (V' is supposed on open circuit), f the frequency and φ the form factor of the primary or secondary potential difference waves, (being the same for both)—the form factor being defined as such that the mean value, of the volts taken positively, is φ times the root mean square of the volts,—then the greatest number (N) of lines of force passing through the core is

$$N = \frac{\varphi}{4}\frac{V}{f\,n} \times 10^8.$$

Of course, if φ is not known we must assume the wave to be a sine wave and

$$\varphi = \frac{2\sqrt{2}}{\pi}.$$

Thus the induction, B, which is N divided by the area of iron in the cross-section of the magnetic circuit, is

$$B = \frac{\varphi}{4}\frac{V\,10^8}{f\,n\,q'\,b\,l}.$$

And the hysteresis loss is $\eta\,B^{1.6}$ ergs per c. c. per cycle.

Thus the total maximum hysteresis loss of power is

H_o = volume of iron × frequency × $\eta B^{1.6}$ ergs per second.

$$= 2 q b l (a + b + c) f \eta \left(\frac{\varphi V 10^8}{4 f n q' b l} \right)^{1.6} \times 10^{-7} \text{ watts.}$$

The mean value ($\overline{H}$) of H is jH_o

$$\therefore \overline{H} = 2 q \eta j f b l (a + b + c) \left(\frac{\varphi V 10^8}{4 f n q' b l} \right)^{1.6} \times 10^{-7} \text{ watts} \quad (3)$$

And if the maximum current density is prescribed, so that σ is known, and

$$n = \frac{p a c}{\sigma}$$

we shall have

$$\overline{H} = 2 q \eta j f b l (a + b + c) \left(\frac{\varphi . V . 10^8 . \sigma}{4 f p q' a c b l} \right)^{1.6} \times 10^{-7} \text{ watts.} \quad (4)$$

The eddy-current loss varies as the square of the induction, as the square of the frequency, as the square of the thickness of the plates, and as the volume of iron. So that if t be the thickness of the plates, we may write the eddy-current loss (E)

$$E = \mathfrak{S} . 2 q b l (a + b + c) t^2 f^2 \left(\frac{\varphi . V . 10^8}{4 f n q' b l} \right)^2 \text{ watts.} \quad (5)$$

Where $\mathfrak{S}$ is a constant.

Dr. Fleming gives the constant $\mathfrak{S} = 10^{-16}$ when t is in mils, the volume in cu. cms. and E in watts.

If the maximum current density is given, so that

$$n = \frac{p a c}{\sigma},$$

we may write

$$E = \mathfrak{S} . 2 q b l (a + b + c) t^2 f^2 \left(\frac{\varphi . V . 10^8 . \sigma}{4 f p q' a c b l} \right)^2 \text{ watts.} \quad (6)$$

XVII. *Applications of the expressions of the last section.*—

We will now apply the results of the last section in the finding of some of the constants used in the former part of the paper.

In § VI, quantities ω, h, and e, are introduced, and equations 1, 3, and 5, of the last section give us

$$\omega = 2\,k\,\rho\,\frac{2\,l + \pi\,\overline{b+c}}{p\,a\,c}\,I^2,$$

$$h = 2\,q\,\eta\,j\,f\,b.\,l.\,(a+b+c)\left(\frac{\varphi.V.10^8}{4\,f\,q'\,b\,l}\right)^{1.6} \times 10^{-7},$$

$$e = 2\,\mathfrak{S}\,q\,b\,l\,(a+b+c)\,t^2 f^2 \left(\frac{\varphi.V.10^8}{4\,f\,q'\,b\,l}\right)^2$$

In § VIII, we are given the proportions of the core. Let a, b, c, and l, be proportional to the dimensions instead of the actual dimensions, then ω_1 h_1 and e_1 are given by the equations 2, 4, and 6, of the last article as follows:

$$\omega_1 = 2\,k\,\rho\,p\,a\,c\,[2\,l + \pi\,\overline{b+c}]\,\delta^2,$$

$$h_1 = 2\,q\,\eta\,j\,f\,b\,l\,(a+b+c)\left[\frac{\varphi.V.10^8.\sigma}{4\,f\,p\,q'\,a\,c\,b\,l}\right]^{1.6} \times 10^{-7},$$

$$e_1 = 2\,\mathfrak{S}\,q\,b\,l\,(a+b+c)\,t^2 f^2 \left[\frac{\varphi.V.10^8.\sigma}{4\,f\,p\,q'\,a\,c\,b\,l}\right]^2.$$

And the number of turns of wire on the primary coil is given by

$$n = \frac{p\,a\,c}{\sigma}\,w^2,$$

where

$$w^{6.4} = 0.8\,\frac{h_1}{\omega_1}$$

In § X, the size of the core is given in terms of quantities H_1, Ω_1, E', and Ω', which can be found from equations 2, 4, and 6, of the last section as follows:

$$\Omega_1 = 2\,k\,\rho\,p\,\delta^2,$$

$$H_1 = 2\,q\,\eta\,j\,f\,\left\{\frac{\varphi\,V\,10^8\,\sigma}{4\,f\,p\,q'}\right\}^{1.6} \times 10^{-7},$$

$$\mathcal{Q}' = 2\,k\,\rho\,p\,\delta^2\,\frac{68}{7}\,\pi\,\sigma^3 = 30.5\,\mathcal{Q}_1\,\sigma^3,$$

$$E' = 2\,\mathfrak{S}\,q\,t^2 f^2 \left\{\frac{\varphi.\,V.\,10^8\,\sigma.}{4\,f\,p\,q'}\right\}^2 \cdot \frac{30}{7}\cdot\frac{9}{7}\cdot\frac{9\pi}{7}\cdot\left[2.\frac{9}{7}\cdot\frac{9\pi}{7}\right]^{-2}\sigma^{-5},$$

$$= 0.413\,\mathfrak{S}\,q\,t^2 f^2 \left\{\frac{\varphi\,V\,10^8\,\sigma}{4\,f\,p\,q'}\right\}^2 \sigma^{-5},$$

where σ is given by

$$\sigma^{5.4} = 0.0138\,\frac{H_1}{\mathcal{Q}_1}.$$

The number of primary turns of wire is given by

$$n = \frac{p\,a\,c}{\sigma},$$

and the number of secondary terms is, of course,

$$n' = n.\,\frac{V'}{V}.$$

Thus we are able to find all the particulars of the transformer in the terms of known quantities.

XVIII. *Similar expressions for the core type of transformer.*—Some small alterations will be required in these results, when the transformer considered is of the core type. Let a, b, c, and l now refer to the dimensions of the transformer shown in Fig. 2.

The volume of iron is now $2\,q\,b\,l\,(a + c + 2\,b)$.

And the mean length of a turn of copper wire is now,

$$2\,l + \pi\left(b + \frac{c}{2}\right).$$

Thus, corresponding to the six equations of § XVI, we get:

$$\overline{\mathcal{Q}} = 2\,k\,\rho\,\frac{2\,l + \pi\,\overline{b + \frac{c}{2}}}{p\,a\,c}\,n^2\,I^2 \qquad \ldots.1$$

$$\overline{\mathcal{Q}} = 2\,k\,\rho\,p\,a\,c\left[2\,l + \pi\,\overline{b + \frac{c}{2}}\right]\delta^2, \qquad \ldots.2$$

$$\overline{H} = 2\,q\,\eta\,j\,f\,b\,l\,(a + c + 2\,b)\left\{\frac{\varphi.\,V.\,10^8}{4\,f.q'.n.b.l}\right\}^{1.6}\times 10^{-7} \ldots . 3$$

$$\overline{H} = 2\,q\,\eta\,j\,f\,b\,l\,(a' + c + 2\,b)\left\{\frac{\varphi.\,V.10^8.\sigma}{4\,f.q'.p.a.c.b.l}\right\}^{1.6}\times 10^{-7}, \quad \ldots . 4$$

$$E = 2\,\mathfrak{E}\,q\,b\,l\,(a + c + 2\,b)\,t^2 f^2\left\{\frac{\varphi.\,V.\,10^8.}{4.f.q'.n.b.l}\right\}^2, \quad \ldots . 5$$

$$E = 2\,\mathfrak{E}\,q\,b\,l\,(a + c + 2\,b)\,t^2 f^2\left\{\frac{\varphi.\,V.10^8\sigma}{4.f.q'.p.a.c\,b.l.}\right\}^2 \quad \ldots . 6$$

Thus the quantities ω, h, and e, used in § VI, become, for a core transformer:

$$\omega = 2\,k\,\rho\,\frac{2\,l + \pi\,\overline{b + \frac{c}{2}}}{p\,a\,c}\,I^2$$

$$h = 2\,q\,\eta\,j\,f\,b\,l\,(a + c + 2\,b)\left(\frac{\varphi.\,V.10^8.}{4\,f\,q'\,b\,l}\right)^{1.6}\times 10^{-7},$$

$$e = 2\,\mathfrak{E}\,q\,b\,l\,(a + c + 2\,b)\left(\frac{\varphi\,V\,10^8}{4\,f\,q'\,b\,l}\right)^2 t^2 f^2.$$

The quantities ω_1, h_1, e_1, of § VIII now become

$$\omega_1 = 2\,k\,\rho\,p\,a\,c\left[2\,l + \pi\,\overline{b + \frac{c}{2}}\right]\delta^2,$$

$$h_1 = 2\,q\,\eta\,j\,f\,b\,l\,(a + c + 2\,b)\left\{\frac{\varphi.\,V.10^8\,\sigma}{4\,f\,q'\,p\,a\,c\,b\,l}\right\}^{1.6}\times 10^{-7},$$

$$e_1 = 2\,\mathfrak{E}\,q\,b\,l\,(a + c + 2\,b)\left\{\frac{\varphi\;V\;10^8.\sigma}{4\,f\,q'\,p\;a\,c\,b\,l}\right\}^2 t^2 f^2,$$

and the number of turns of wire on the primary coil is given by

$$n = \frac{p\,a\,c}{\sigma}\,x^2, \qquad \text{where } x^{6.4} = 0.8\,\frac{h_1}{\omega_1}.$$

In this, it must be remembered that a, b, c, l, are proportional to the dimensions, instead of being the actual dimensions.

We next come to the core transformer of maximum efficiency discussed in § XI.

Here

$$H_1 = 2\,q\,\eta\,j\,f\,\left\{\frac{\varphi.\,V.\,10^8.\,\sigma}{4\,f\,q'\,p}\right\}^{1.6} \times 10^{-7},$$

$$\mathcal{Q}_1 = 2\,k\,\rho\,p\,\delta^3,$$

$$\mathcal{Q}' = 2\,k\,\rho\,p\,\delta^3 \times \frac{34}{7}\,\pi\,c^3 = 15.25\,\mathcal{Q}_1\,c^3,$$

$$E' = 2\,\mathfrak{S}\,q\,t^2\,f^2\left\{\frac{\varphi.\,V.10^8.\sigma}{4.f\,q'\,p}\right\}^2 \times \frac{9}{14}\cdot\frac{9\pi}{14}\cdot\frac{60}{14}\left\{\frac{9}{14}\cdot\frac{9\pi}{14}\cdot 2\right\}^{-2} c^{-8},$$

$$= 1.65\,\mathfrak{S}\,q\,t^2\,f^2\left\{\frac{\varphi\,V\,10^8\,\sigma}{4\,f\,q'\,p}\right\}^2 c^{-5}.$$

Where c is given by

$$c^{5.4} = 0.0633\,\frac{H_1}{\mathcal{Q}_1}$$

The number of turns of wire in the primary coil is given by

$$n = \frac{p\,a\,c}{\sigma},$$

and the number of turns in the secondary is of course

$$n' = n\,\frac{V'}{V}.$$

Thus the transformer of maximum efficiency is completely determined. (See also Appendix B).

XIX. *Remarks on some of the constants.*—Since the dimensions and particulars of the transformer designed as above are given by an equation of the form,—

$$c^{5.4} = \text{a known constant},$$

it follows that a small error in this constant will produce an error of only about a sixth of the amount in the linear dimensions; and, since the energy wasted is a minimum, a small error in any dimension will produce but a very small increase in this waste of energy.

In the expression for $c^{5.4}$, the constant involved to the highest power is that which we have called p, and this is raised to the

2.6th power. There should, however, be no difficulty in determining p, and, if care be taken to so choose it that there is plenty of room for insulation, it can hardly be said to be subject to error, as there is never any harm in putting a little more insulation in.

Some of the constants, such as k and φ, may not be determinable with any degree of accuracy, but they are worth determining as nearly as possible when the highest efficiency is sought. The difference in the power factor φ for a peaked wave, and for a square wave, for instance, may make as much as 20 per cent. difference in the iron losses.

The constant η, in the above formulæ, should be such that η times the 1.6th power of the maximum induction represent the hysteresis loss, in ergs per cu. cm. per cycle, at something like the right induction. I have preserved 1.6 as the index throughout as giving results sufficiently near for most practical purposes, and in order to obtain the definiteness given by numerical results as far as possible. Nevertheless, the index usually seems somewhat less than 1.6. Thus Ewing and Klassen[1] came across a specimen of transformer iron in which the index was about 1.475 over the range of induction usual in this work. Accordingly the results of this paper are worked out for a general value of the index in Appendices B. and C., and tables are giving to facilitate the calculations for different values of the index.

XX. *Comparison with actual transformers.*—We will now show how transformers, designed according to the foregoing methods, compare with those at present in use. With this view, we will obtain the dimensions and particulars of transformers, having the same output, and working under the same conditions of voltage and frequency, as two of the transformers at the Central Technical College. Moreover, such constants as we want at starting, we will take precisely the same as obtain in these transformers, and, since the conditions of variations of load, under which the transformers were intended to work are not known, we will assume, if possible, that the transformers in question, are designed to best suit their all day load, that is, as shown in § IV, that the conditions of load are such that the average value of the ohmic loss, is equal to 0.8 times the average value of the hysteresis loss, together with the average value of the

1. Phil. Trans. Vol. 184, p. 1019.

eddy-current loss; and we will satisfy the same conditions of load in the transformers designed.

We will consider first, a five kilowatt Westinghouse transformer of recent type, and of the shell pattern. The particulars of the transformer are as follows:

Primary Voltage = 2,000.
Secondary Voltage = 100.
Frequency = 100.

Core

Thickness of plates (t) = 13.3 mils.
Thickness of insulation = 1.7 mils.
Hence $q = q' = 0.89$. (§ xv).
Number of plates 400.
Length of core (l Fig. 1) = 6 inches.
Breadth of core (b Fig. 1) = 3¼ inches.
Winding space ($a \times c$ Fig. 1) = 5¾ × 3¼ square inches.
Volume of iron = 467 cubic inches, approximately.
Weight of iron = 130 lbs., approximately.

Coils

Primary, resistance 7.32 ohms.
Secondary, resistance 0.02 ohms.
Number of secondary turns = 48.
Mean length of a turn = 34 inches.
Area of cross-section of wire = 0.0595 sq. ins. = 0.383 sq. cm. Hence maximum current density = 840 amps. per sq. inch.
= 130.5 amps. per sq. cm.
Hence p,—the ratio of the total cross-section of copper in a coil, to the area of the winding pocket, is

$$\frac{48 \times 0.0595}{5.75 \times 3.25} = 0.153 \qquad \text{(§ xiv.)}$$

The maximum induction is 3900 lines per sq. cm.
The losses appear to be proportioned as follows:

Maximum ohmic loss, 100 watts.
Maximum hysteresis loss, 79.5 watts.
Maximum eddy-current loss, 20.5 watts.

Hence the hysteretic constant

$$\eta = 0.00187.$$

and the eddy loss constant

$$\mathfrak{E} = 10^{-16}.$$

We are assuming that the mean ohmic loss is equal to 0.8 times the mean hysteresis loss, together with the eddy-current loss, and, assuming the mean hysteresis loss is 0.99 of the maximum, and the mean ohmic loss is k times the maximum, we have

$$k \times 100 = 0.8 \times 0.99 \times 79.5 + 20.5.$$

Whence $k = 0.835$, and $j = 0.99$. (§ XIII.)

Hence, using the same notation as in §§ X and XVII we have (See § XVII).

$$\mathcal{Q}_1 = 2 \times 0.835 \times 1.85 \times 10^{-6} \times 0.153 \times 130.5^2,$$

$$= 8.04 \times 10^{-3},$$

$$H_1 = 2 \times 0.89 \times 0.00187 \times 0.99 \times 100 \left\{ \frac{100\sqrt{2} \times 10^8 \times 0.383}{2\pi \times 100 \times 0.153 \times 0.89} \right\}^{1.6} \times 10^{-7},$$

$$= 9.98 \times 10^4.$$

$$\therefore c^{6.4} = 0.0138 \frac{9.98}{8.04} \times 10^7 = 1.714 \times 10^5,$$

$$c = 6.57 \text{ cms.}$$

Again

$$E' = 0.413 \times 10^{-16} \times 0.89 \times 13.3^2 \times 10^4 \left\{ \frac{100\sqrt{2} \times 10^8 \times 0.383}{2\pi \times 100 \times 0.153 \times 0.89} \right\}^2 c^{-6},$$

$$\mathcal{Q}' = 30.5\ \mathcal{Q}_1\ c^3,$$

$$\therefore \frac{E'}{\mathcal{Q}'} = 0.31.$$

Hence

$$1 + 0.69 \frac{E'}{\mathcal{Q}'} = 1.21,$$

$$1 - 0.036 \frac{E'}{\mathcal{Q}'} = 0.989,$$

and the corrected values of l, a, b, and c, are:—

$$l = \frac{9}{7}\pi \times 6.57 \times 1.21 = 32.1 \text{ cms.}, = 12.6 \text{ inches.}$$

$$b = \frac{9}{7} \times 6.57 \times 1.21 = 10.21 \text{ cms.}, = 4.02 \text{ inches.}$$

$$a = 2 \times 6.57 \times 0.989 = 13.00 \text{ cms.}, = 5.12 \text{ inches.}$$

$$c = 6.57 \times 0.989 = 6.5 \text{ cms.}, = 2.56 \text{ inches.}$$

Hence the number of turns in the low pressure coil is

$$n = \frac{0.153 \times 2 \times 6.50^2}{0.383},$$

$$= 34$$

The volume of iron is

$$2 \times 0.89 \times 4.02 \times 12.6 \times 11.7 \text{ cu. inches,}$$
$$= 1054 \text{ cu. inches.}$$

The weight of iron is

294 lbs.

The maximum ohmic loss is

$$2 \times 1.85 \times 10^{-6} \times 0.153 \times 13.0 \times 6.5 \times 116.7 \times 130.5^2,$$

$$= 94.8 \text{ watts.}$$

The mean ohmic loss is $94.8 \times 0.835 = 79.1$ watts.

The maximum induction in the iron is

$$\frac{10^8 \sqrt{2} \times 0.383}{2\pi \times 0.89 \times 0.153 \times 13 \times 6.5 \times 32.1 \times 10.21},$$

$$= 2290 \text{ lines per sq. cm.}$$

The eddy-current loss is

$$E = 10^{-16} \times \text{volume of iron} \times t^2 \times f^2 \times B^2,$$

$$= 10^{-16} \times 1054 \times 16.39 \times 13.3^2 \times 10^4 \times 2290^2,$$

$$= 16.0 \text{ watts.}$$

The maximum hysteresis loss is

$$H = 0.00187 \times 1054 \times 16.39 \times 100 \times [2290]^{1.6} \times 10^{-7},$$

$$= 76.8 \text{ watts.}$$

Hence the total loss at full load is,—

$$94.8 + 76.0 + 16.0 \text{ watts},$$

$$= 187 \text{ watts}.$$

against 199 watts in the Westinghouse transformer:—a saving of 12 watts, or about 6 per cent. of the total loss.

The mean loss is.

$$(79.1 + 0.99 \times 76.8 + 16) \text{ watts},$$

$$= 171.1 \text{ watts}.$$

Against $0.835 \times 100 + 0.99 \times 79.5 + 20.5$ watts

$$= 182.5 \text{ watts in the Westinghouse},$$

a saving of 11.4 watts or $6\frac{1}{4}$ per cent of the total loss.

Thus, a transformer designed according to the principles here advocated wastes 11 or 12 watts less power than the Westinghouse transformer in question. It, however, requires more than twice the weight of iron, and about 5 per cent. less copper than the latter. (The amount of copper varies as the ohmic loss when the current density is given).

It must be said, however, that by adopting constants which hold in a transformer of very different proportions, we rather handicap ourselves. Thus, whilst we have less heat to dissipate, we have a greater radiating surface, and so might do with less ventilation space among the coils, and so with a larger value for p. This would reduce both the size of the transformer and its losses.

As shown in Appendix A, the losses could be reduced, without altering the size, by making the core plates thinner.

The thickness, for the maximum efficiency is, in this case, about 7 mils, (assuming the same thickness of core insulation, 1.7 mils). With this thickness, using the same constants as above, except that

$$q = \frac{7}{8.7} = 0.805$$

we shall have, just as before

$$c^{8.4} = 1.82 \times 10^5,$$

$$c = 6.64 \text{ cms}.$$

Again

$$\frac{E'}{Q'} = 0.087,$$

$$\therefore 1 + 0.69\,\frac{E'}{Q'} = 1.060, \qquad 1 - 0.036\,\frac{E'}{Q'} = 0.997,$$

$l = 28.4$ cms., $= 11.18$ inches.
$a = 13.24$ cms., $= 5.20$ inches.
$b = 9.05$ cms., $= 3.56$ inches.
$c = 6.62$ cms., $= 2.60$ inches.

The volume of iron is 726 cu. inches; and its weight is 202 lbs.

The induction is 3130 lines per sq. cm.

The number of turns of wire on the low pressure coil is 35.

The maximum ohmic loss is 89.5 watts.

The eddy-current loss is 5.7 watts.

The maximum hysteresis loss is 87.4 watts.

The maximum total loss is

$$89.5 + 86.5 + 5.6 \text{ watts.}$$
$$= 181.6 \text{ watts.}$$

thus wasting $17\frac{1}{2}$ watts, ($8\frac{3}{4}\%$) less than the Westinghouse transformer.

The mean loss is 167 watts,—15.5 watts less than the Westinghouse apparatus.

Thus we see that we save about $8\frac{3}{4}$ per cent. of the energy but our transformer has 202 lbs. of iron instead of 130 lbs. and 55 lbs. of copper instead of 61 lbs. We should however expect the transformer so designed to keep cooler than the Westinghouse as it has less heat to dissipate and a greater radiating surface. Thus it will better stand an overload. Its regulation, moreover, should be better, for the same subdivision of the coils, both because the copper losses are less, and that the magnetic leakage is very much smaller.

We shall next find the particulars of a transformer, designed as above, and having, as far as possible, the same constants as a certain "Mordey Victoria" transformer at the Central Technical College.

The transformer in question is of the shell pattern and intended to convert three kilowatts from a pressure of 2,000 volts,

to a pressure of 100 volts. The particulars of it are as following:—

Core

Thickness of core plates = 15 mils.
Ratio of volume of iron to total volume of core = 0.89 (= q).
Thickness of insulation between plates = 1.85 mils.
Length of core (l Fig. 1) = 52 cms. = 20½ inches.
Breadth of magnetic circuit (b Fig. 1) = 7.62 cms. = 3 inches.
Winding space ($a \times c$ Fig. 1) = 7.62 × 3.81 sq. cms. = 3 × 1½ sq. ins.
Volume of iron = 820 cubic inches, } approximately.
Weight of iron = 230 lbs. }
Hysteresis constant, (η), (with index 1.6) = 0.0020.
Eddy-current constant (δ) = 0.95×10^{-16}.

Coils.

Primary, 456 convolutions, 8.8 ohms resistance.
Secondary, 24 convolutions, 0.083 ohms resistance.
Cross-section of strips of secondary coils = 0.0316 sq. ins. = 0.204 sq. cms.
Hence maximum current density = 950 amps. per sq. inch,
= 147 amps. per sq. cms.,
and ratio of cross-section of copper in the secondary coil to the total area of winding pocket (p) = 0.169.
The total weight of copper is about 29 lbs.
The normal frequency is 100, and we shall assume the applied P. D. to follow a sine law. Thus $\phi = 2\sqrt{2} \div \pi$.

In the Mordey transformer the induction, when working normally, is 2670 lines per sq. cm.

The maximum ohmic loss is 60 watts.
The maximum hysteresis loss is 81.5 watts.
The maximum eddy-current loss is 20.5 watts.

Thus we see that it is impossible to make the mean ohmic loss equal to 0.8 times the hysteresis loss, together with the eddy-current loss, as the maximum ohmic loss is already less than the mean should be. I shall assume, therefore, that the transformer has been designed to work always at full load, this being as favorable as possible to the Mordey transformer.

Hence $k = 1$,

and we will assume $j = 0.985$. (§ XIII).

Thus we have (§ XVII).

$$\mathcal{Q}_i = 2 \times 1.85 \times 10^{-6} \times 0.169 \times 147^2,$$

$$= 1.35 \times 10^{-2}.$$

$$H_1 = 2 \times 0.89 \times 0.002 \times 0.985 \times 100 \left\{ \frac{\sqrt{2} \times 10^5 \times 0.204}{2\pi \times 0.169 \times 0.89} \right\}^{1.6} \times 10^{-7},$$

$$= 3.31 \times 10^4.$$

$$c^{6.4} = 0.0138 \frac{H_1}{Q_1},$$

$$= 3.381 \times 10^4;$$

$$\therefore c = 5.10 \text{ cms.}$$

$$E' = 7.31 \times 10^4 c^{-5},$$

$$Q' = 4.12 \times 10^{-1} c^3,$$

$$\frac{E'}{Q'} = 0.394,$$

$$1 + 0.69 \frac{E'}{Q'} = 1.271,$$

$$1 - 0.036 \frac{E'}{Q'} = 0.986.$$

Hence the corrected values of the linear dimensions are:

$l = 26.1^*$ cms., $= 10.25$ inches.
$b = 8.31$ cms., $= 3.27$ inches.
$a = 10.02$ cms., $= 3.95$ inches.
$c = 5.01$ cms., $= 1.97$ inches.

Volume of iron

$$= 2\, q.\, b.\, l.\, (a + b + c),$$
$$= 550 \text{ cu. inches.}$$

Weight of iron = 153 lbs.

Volume of copper.

$$= 2\, p\, a\, c\, (2\, l + \pi\, \overline{b + c}),$$

$$= 97.1 \text{ cubic inches.}$$

Weight of copper = 30.9 lbs.

The number of turns of wire in the secondary coil is

$$n = \frac{p\,a\,c}{\sigma} = \frac{0.169 \times 10.02 \times 5.01}{0.204} = 42.$$

Maximum ohmic loss (§ xvi).

$$1.85 \times 10^{-6} \times 97.1 \times 16.39 \times 147^2,$$
$$= 63.6 \text{ watts.}$$

The maximum induction = 2810 lines per sq. cm.
Hence

Maximum hysteresis loss = 59.3 watts.

Eddy-current loss = 15.2 watts.

Hence the mean total loss

$$= 63.6 + 58.4 + 15,$$
$$= 137 \text{ watts.}$$

against 160.5 watts in the Mordey transformer.

Thus there is a saving of 23½ watts by designing the transformer so as to have a maximum efficiency. This is accompanied by an increase of weight of copper of nearly 2 lbs. and a decrease of weight of iron of about 77 lbs.

We will next find what the particulars of the transformer would have been, if the plates had been 7½ mils thick,—which thickness about gives minimum iron loss, with the given thickness, (1.85 mils), of core insulation. (Appendix A). This thickness of core plate gives $q = 0.803$.

We will also assume, (which is true for the iron of the Mordey transformer over a considerable range of induction), that the hysteresis loss per cubic cm. per cycle is equal to 0.00251 times the 1.57th power of the induction. (Appendix B).

Thus

$$\frac{H_1}{Q_1} = 2.94 \times 10^{-6} (3.39 \times 10^7)^{1.57}, \qquad \text{(§ xvii)}$$

$$c^{6.28} = 0.0165 \frac{H_1}{Q_1},$$

$$= 4.85 \times 10^{-8} (3.39 \times 10^7)^{1.57},$$

$$c = (4.85 \times 10^{-8})^{\frac{1}{6.28}} \times (3.39 \times 10^7)^{\frac{1}{4}},$$

$$= 5.22 \text{ cms.}$$

$$\therefore b = 6.24 \text{ cms.,}$$—[Table I, Appendix B.]

$$a = 10.45 \text{ cms.,}$$

$$l = 19.60 \text{ cms.}$$

These are the first approximations to a, b, c, and l, uncorrected for the effects of eddy-currents.

Again

$$E' = 5.9 \text{ watts,}$$

and

$$Q' = 55.4 \text{ watts.}$$

$$\therefore \frac{E'}{Q'} = 0.107,$$

and

$$1 + 0.75 \frac{E'}{Q'} = 1.08, \quad 1 - 0.047 \frac{E'}{Q'} = 0.995, \quad \text{[Appendix C].}$$

Hence the corrected values of the dimensions are:

$$a = 10.45 \times 0.995 = 10.40 \text{ cms.,} = 4.09 \text{ inches,}$$

$$c = 5.22 \times 0.995 = 5.20 \text{ cms.,} = 2.04 \text{ inches.}$$

$$b = 6.24 \times 1.08 = 6.73 \text{ cms.,} = 2.65 \text{ inches.}$$

$$l = 19.60 \times 1.08 = 21.20 \text{ cms.,} = 8.35 \text{ inches.}$$

Hence the volume of iron

$$= 0.803 \times 2 \times 2.65 \times 8.35 \times 8.78 \text{ cu. ins.,}$$

$$= 312 \text{ cubic inches.}$$

Weight of iron = 87 lbs.

Induction of iron

$$= \frac{3.39 \times 10^7}{10.4 \times 5.2 \times 6.73 \times 21.2} = 4400 \text{ lines per sq. cm.}$$

Maximum hysteresis loss

$$= 0.00251 \times 312 \times 16.39 \times 100 \times (4400)^{1.57} \times 10^7,$$

$$= 67.2 \text{ watts.}$$

Eddy-current loss.

$$= 0.95 \times 10^{-16} \times 312 \times 16.39 \times 10^4 \times 7.5^2 \times 4400^2,$$

$$= 5.3 \text{ watts.}$$

Maximum ohmic loss

$$= 2 \times 1.85 \times 10^{-6} \times 0.169 \times 10.40 \times 5.20 \times 79.9 \times 147^2,$$

$$= 58.3 \text{ watts.}$$

Hence the full load total loss is

$$58.3 + 66.2 + 5.2 = 129.7 \text{ watts, against}$$

160.5 watts in the Mordey transformer.

Volume of copper 89.1 cubic inches.

Weight of copper 28.4 lbs.

Number of turns of wire in secondary coil is

$$n = \frac{0.169 \times 5.2 \times 10.4}{0.204} = 45.$$

Thus this transformer, running always at full load, wastes 31 watts, (nearly 20 per cent.) less power than the Mordey, whilst there is almost 1 lb. less copper, and about 143 lbs. less iron used in its construction. The Mordey transformer in fact, can claim an advantage in only one important particular, viz.:—that its radiating surface is proportionately larger, so that it should keep cooler. Had we taken a lower maximum current density, however, we should have increased the surface, and diminished the wasted energy, and so we could have designed a transformer better than the Mordey in every particular of moment.

It may be mentioned that the Mordey transformer is an older apparatus than the Westinghouse, and the type has doubtless been improved since the time of its manufacture.

The following table exhibits the particulars of the transformers considered in convenient form, and enables a comparison to be made between them without difficulty. They are numbered 1, 2, 3, 4, 5, 6, in the order in which they are discussed above.

TRANSFORMER.	1.	2.	3.	4.	5.	6.
Output in kilowatts	5	5	5	3	3	3
Primary P. D., (V) in volts	2000	2000	2000	2000	2000	2000
Secondary P. D., (V') in volts	100	100	100	100	100	100
Frequency of alternations, (f) in periods per second	100	100	100	100	100	100
Power factor of primary P. D wave (φ)	$2\sqrt{2}+\pi$	$2\sqrt{2}+\pi$	$2\sqrt{2}+\pi$	$2\sqrt{2}+\pi$	$2\sqrt{2}+\pi$	$2\sqrt{2}+\pi$
Maximum secondary current, (I') in amperes	50	50	50	30	30	30
Cross-section of secondary wire, (σ) in sq. cms	0.383	0.383	0.383	0.204	0.204	0.204
Maximum current density, (δ) in amperes per sq. cm.	130.5	130.5	130.5	147	147	147
Number of convolutions (n') in the secondary	48	34	35	24	42	45
Value adopted for specific resistance of copper, (ρ) microhms per cm. cube	1.85	1.85	1.85	1.85	1.85	1.85
Ratio of volume of *wire* in secondary to total winding space (ϕ)	0.153	0.153	0.153	0.169	0.169	0.169
Ratio of mean to maximum ohmic loss, (k)	0.835	0.835	0.835	1	1	1
Ratio of mean to maximum hysteresis loss, (γ)	0.99	0.99	0.99	0.985	0.985	0.985
Hysteretic constant, (η)	0.00187	0.00187	0.00187	0.0020	0.0020	0.00251
Index occurring in Steinmetz formula for hysteresis loss, (ε)	1.6	1.6	1.6	1.6	1.6	1.57
Eddy-current constant, ($\mathfrak{E}$)	10^{-16}	10^{-16}	10^{-16}	0.95×10^{-16}	0.95×10^{-16}	0.95×10^{-16}
Thickness of core plates, (t) in mils	13.3	13.3	7	15	15	7.5
Thickness of insulation between core plates, (t') in mils	1.7	1.7	1.7	1.85	1.85	1.85
Ratio of volume of iron to that of core, ($q=t+t'+t'$)	0.89	0.89	0.805	0.89	0.89	0.803
l, (Fig. 1) in inches	6	12.6	11.18	20.5	10.25	8.35
a, (Fig. 1) in inches	5.75	5.12	5.20	3	3.95	4.09
b, (Fig. 1) in inches	3.5	4.00	3.56	3	2.27	2.65
c, (Fig. 1) in inches	3.25	2.56	2.60	1.5	1.97	2.04
Maximum ohmic loss, (Q) in watts	100	94.8	89.5	60	63.6	58.3
Maximum hysteresis loss, (H), in watts	79.5	76.8	87.4	82.5	59.3	67.2
Maximum eddy-current loss, (E), in watts	20.5	16.0	5.7	20.5	15.2	5.3
Maximum total loss, in watts	199	187	181.6	160.5	137	129.7
Mean total loss, in watts	182.5	171.1	167	160.5	137	129.7
Magnetic induction in iron, in lines per sq. cm	3900	2290	3130	2670	2810	4400
Weight of copper, in lbs	61.5	58.5	55	29.2	30.9	28.4
Weight of iron, in lbs	130	294	262	230	253	87
Drop of secondary voltage at full load, due to copper losses only, in volts	2	1.9	1.79	2	2.12	1.95
Radiating surface per watt wasted, at full load, in sq. inches	3.75	5.3	4.9	5.0	4.6	3.85

APPENDIX A.

The Best Thickness of Core Plates

Wherever we have had occasion to discuss the effects of eddy-currents on our design, we have assumed the energy wasted by them to be small compared with the other losses. But as, in many cases, the eddy-current loss of energy is a considerable fraction of the total iron loss it will not perhaps be thought out of place to discuss whether it is better so than small.

Professors J. J. Thomson and Ewing[1] have discussed the subject of the energy wasted by the eddy-currents in transformer plates, and included the effect on the hysteresis loss of the screening action of the eddy-currents.

They conclude that this latter action is practically negligible when the plates are no more than 20 mils thick, and as transformer plates are usually thinner than this, we shall neglect it in our investigations. The professors show the value of effective lamination, and Professor Ewing points out that the limit of thinness is determined by the consideration that, if the plates are too thin, the core will either have to be larger or to be worked at a higher induction than is consistent with small hysteresis loss.

The following investigation is made with a view to determine the thickness of plate which will make the total iron loss a minimum. We shall assume that the core has a definite size and shape, and that the thickness (t') of insulation between core plates is given, as also the total number of lines of force (N) in the magnetic circuit.

Let v be the volume of the core, including insulation, A the area of cross-section of the magnetic circuit, also including insulation, f the frequency, and t the thickness of the core plates.

Then the volume of iron is $\frac{t}{t+t'}\,v$,

The area of the iron portion of the cross-section of the magnetic circuit is $\frac{t}{t+t'}\,A$.

The induction in the iron is $\frac{N}{\frac{t}{t+t'}\,A}$

Hence the hysteresis loss is

$$\eta\,\frac{t}{t+t'}\,v.f.\left[\frac{N}{\frac{t}{t+t'}\,A}\right]^{1.6}\times 10^{-7}\text{ watts},$$

1. See *London Electrician*, volume xxviii.

and the eddy-current loss is

$$\mathcal{E}\,\frac{t}{t+t'}\,v\,f^2\,t^2\left[\frac{N}{\frac{t}{t+t'}\,A}\right]^2 \text{ watts.}$$

Putting $\frac{t}{t+t'} = q$, as in § xv

$$t = \frac{q}{1-q}\,t',$$

the total iron loss is

$$f\,v\left\{\eta\,q\left(\frac{N}{q\,A}\right)^{1.6}\times 10^{-7} + \mathcal{E}\,q f\,\frac{q^2\,t'^2}{(1-q)^2}\left(\frac{N}{q\,A}\right)^2\right\} \text{ watts.}$$

Thus, if q is such as to make the iron loss a minimum.

$$-\,0.6\,\eta\,q^{-1.6}\,10^{-7} + \mathcal{E}\,f\,\frac{q}{(1-q)^2}\,t'^2\,B^{0.4}\left[\frac{1}{q}+\frac{2}{1-q}\right]=0,$$

$$\text{where } B = \frac{N}{A},$$

the average induction through the area A.

Thus

$$0.6\,\eta\,q^{-1.6}\,10^{-7} = \mathcal{E}\,f\,t'^2\,B^{0.4}\,\frac{1+q}{(1-q)^3},$$

or

$$\frac{(1-q)^3}{q^{1.6}(1+q)} = \frac{\mathcal{E}}{0.6.\eta}\,f\,t'^2\,B^{0.4}\,10^7.$$

This gives q

We shall best find q by plotting a curve having q as ordinate, and

$$\frac{(1-q)^3}{q^{1.6}(1+q)}$$

as abscissa, (Fig. 4) and obtaining q from the curve.

With ordinary values for the constants we shall find that q is less than the value which usually obtains, and except for low frequencies, t is usually less than the 10 mils which appears to be about the inferior limit of thickness at present used.

Thus if

$$\mathfrak{E} = 10^{-16}, \quad f = 100, \quad \eta = 0.002,$$

$$B = 2500, \quad t' = 3 \text{ mils},$$

$$\frac{(1-q)^3}{q^{1.6}(1+q)} = \frac{10^{-16} \times 100 \times 3^2 \times (2500)^{0.4} \times 10^7}{0.6 \times 0.002},$$

$$= 0.01706.$$

Hence from the curve

$$q = 0.737,$$

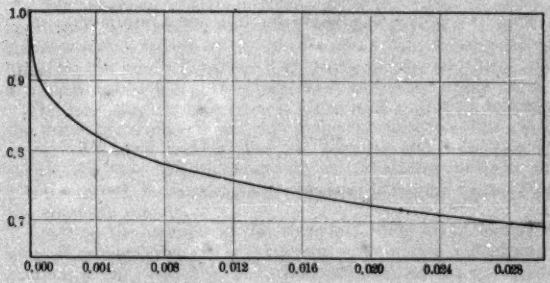

Fig. 4

thus

$$1 - q = 0.263,$$

and

$$t = \frac{737}{263} \times 3 \text{ mils},$$

$$= 8.4 \text{ mils}.$$

The induction is

$$\frac{2500}{0.737} = 3400 \text{ lines per sq. cm.}$$

Thus we see that both q and t are smaller than is usual.

If $t' = 2$ mils

$$\frac{(1-q)^3}{q^{1.6}(1+q)} = 0.00759.$$

Whence

$$q = 0.79,$$

$$1 - q = 0.21,$$

$$t = \frac{79}{21} \times 2 \text{ mils.}$$

$$= 7.5 \text{ mils.}$$

The induction is

$$\frac{2500}{0.79} = 3170 \text{ lines per sq. cm.}$$

Again suppose

$$B = 2000, \quad f = 50, \quad t' = 1.5 \text{ mils},$$

$$\mathfrak{S} = 10^{-16}, \quad \eta = 0.002,$$

then

$$\frac{(1-q)^3}{q^{1.6}(1+q)} = 0.00196,$$

$$\therefore q = 0.857,$$

$$1 - q = 0.143,$$

$$t = \frac{857}{143} \times 1.5 \text{ mils} = 8.9 \text{ mils.}$$

And the induction is

$$\frac{2000}{0.857} = 2340 \text{ lines per sq. cm.}$$

t is of course the thickness of iron, and t' the distance between the iron of consecutive plates, so that it should include a little for rugosities of surface.

It thus appears that, with the same sized core and the same

total flux, the diminution of hysteresis loss due to using plates, say 14 mils thick as against thinner ones, does not balance the increase of eddy-current loss, and to ensure minimum iron loss, the plates should be considerably thinner than is usual at present.

APPENDIX B.

Calculations for the Transformer of Maximum Efficiency

We will here give a more extended treatment of the solution of the equations of § x, in which, however, we will assume that the hysteresis loss varies as the εth power of the induction, in order to see how the proportions obtained depend on the index of this power.

The problem is, to make $\mathcal{Q} + H + E$ a minimum, subject to the condition

$$\mathcal{Q} = \frac{\varepsilon}{2} H + E,$$

where

$$\mathcal{Q} = \mathcal{Q}_1 a c (2 l + \pi \overline{b + c}),$$

$$H = H_1 (a + b + c) (a c)^{-\frac{\varepsilon}{2}} (b l)^{-\varepsilon + 1},$$

$$E = E_1 (a + b + c) (a c)^{-\frac{\varepsilon}{2}} (b l)^{-1},$$

Thus, if W be the total energy wasted,

$$W = \mathcal{Q} + H + E,$$

$$= \mathcal{Q} + H + E + K(\mathcal{Q} - \frac{\varepsilon}{2} H - E),$$

$$= \mathcal{Q}(1 + K) + H(1 - \frac{\varepsilon}{2} K) + E(1 - K)$$

Thus

$$\frac{\partial W}{\partial l} = \frac{2}{2 l + \pi \overline{b + c}} (1 + K) \mathcal{Q} + \frac{1 - \varepsilon}{l} \left(1 - \frac{\varepsilon}{2} K\right) H$$

$$- \frac{1}{l} (1 - K) E = 0,$$

or

$$\frac{2}{2\,l+\pi\sqrt{b+c}}(1+K)\,Q=\frac{\varepsilon-1}{l}\left(1-\frac{\varepsilon}{2}K\right)H + \frac{1}{l}(1-K)\,E.$$

$$\frac{\delta W}{\delta a}=\frac{1}{a}(1+K)\,Q+\left(\frac{1}{a+b+c}-\frac{\varepsilon}{a}\right)\left(1-\frac{\varepsilon}{2}K\right)H + \left(\frac{1}{a+b+c}-\frac{2}{a}\right)(1-K)\,E=0$$

or

$$\frac{1}{a}(1+K)\,Q=\frac{\varepsilon\sqrt{b+c}+\sqrt{\varepsilon-1}\,a}{a\,(a+b+c)}\left(1-\frac{\varepsilon}{2}K\right)H + \frac{2\sqrt{b+c}+a}{a\,(a+b+c)}(1-K)\,E$$

$$\frac{\delta W}{\delta b}=\frac{\pi}{2\,l+\pi\sqrt{b+c}}(1+K)\,Q+\left(\frac{1}{a+b+c}-\frac{\varepsilon-1}{b}\right)\left(1-\frac{\varepsilon}{2}K\right)H+\left(\frac{1}{a+b+c}-\frac{1}{b}\right)(1-K)\,E=0,$$

or

$$\frac{\pi}{2\,l+\pi\sqrt{b+c}}(1+K)\,Q=\frac{(\varepsilon-1)(a+c)-(2-\varepsilon)\,b}{b\,(a+b+c)}\left(1-\frac{\varepsilon}{2}K\right)H+\frac{a+c}{b\,(a+b+c)}(1-K)\,E.$$

$$\frac{\delta W}{\delta c}=\left(\frac{\pi}{2\,l+\pi\sqrt{b+c}}+\frac{1}{c}\right)(1+K)\,Q+\left(\frac{1}{a+b+c}-\frac{\varepsilon}{c}\right)\left(1-\frac{\varepsilon}{2}K\right)H+\left(\frac{1}{a+b+c}-\frac{2}{c}\right)(1-K)\,E=0,$$

or

$$\frac{2\,l+\pi\sqrt{b+2\,c}}{c\,(2\,l+\pi\sqrt{b+c})}(1+K)\,\mathcal{Q}=\frac{\varepsilon\sqrt{a+b}+\sqrt{\varepsilon-1}\,c}{c\,(a+b+c)}$$

$$\left(1-\frac{\varepsilon}{2}K\right)H+\frac{2\,(a+b)+c}{c\,(a+b+c)}(1-K)E.$$

Neglecting E' for the time, these four equations can be written:

$$\frac{(1+K)\,\mathcal{Q}}{\left(1-\frac{\varepsilon}{2}K\right)H}=\frac{(\varepsilon-1)(2\,l+\pi\sqrt{b+c})}{2\,l}=\frac{\varepsilon\sqrt{b+c}+\sqrt{\varepsilon-1}.a}{a+b+c}$$

$$=\frac{(2\,l+\pi\sqrt{b+c})\,(\sqrt{\varepsilon-1}\,\sqrt{a+c}-\sqrt{2-\varepsilon}\,b)}{\pi\,b\,(a+b+c)}$$

$$=\frac{(2\,l+\pi\sqrt{b+c})\,(\varepsilon\sqrt{a+b}+\sqrt{\varepsilon-1}\,c)}{(2\,l+\pi\,b+2\,c)\,(a+b+c)}$$

If for shortness, we write these

$$\frac{(1+K)\,\mathcal{Q}}{\left(1-\frac{\varepsilon}{2}K\right)H}=[1]=[2]=[3]=[4],$$

from $[1]=[3]$ we get

$$l=\frac{\varepsilon-1}{2}\pi\,\frac{b\,(a+b+c)}{(\varepsilon-1)\,(a+c)-\sqrt{2-\varepsilon}\,b}, \qquad (1)$$

Substituting for l in the equation $[1]=[4]$ we get

$$(\varepsilon-1)\,b\,(a+b+c)+(b+2\,c)\,[(\varepsilon-1)\,(a+c)-\sqrt{2-\varepsilon}\,b]$$

$$=b\,(\varepsilon\sqrt{a+b}+\sqrt{\varepsilon-1}\,c),$$

or

$$2\sqrt{\varepsilon-1}.c.\sqrt{a+c}=b\,(\sqrt{2-\varepsilon}\,a+3-\varepsilon\,b+5-3\,\varepsilon\,c). \qquad (2)$$

Again, from $[1]=[2]$ we get, substituting for l from 1,

$$\frac{(\varepsilon-1)\,\pi\,\{\sqrt{\varepsilon-1}\,b\,(a+b+c)+(b+c)\,(\sqrt{\varepsilon-1}\,\sqrt{a+c}-\sqrt{2-\varepsilon}\,b)\}}{(\varepsilon-1)\,\pi\,b\,(a+b+c)}$$

$$=\frac{\varepsilon\sqrt{b+c}+\sqrt{\varepsilon-1}\,a}{a+b+c},$$

or

$$(\varepsilon - 1)\, b\, (a + b + c) + (b + c)\, (\overline{\varepsilon - 1}\ \overline{a + c} - \overline{2 - \varepsilon}\ b)$$
$$= b\, (\varepsilon \overline{b + c} + \overline{\varepsilon - 1}\ a),$$

or

$$\{ (\varepsilon - 1)\, (a + c) - (3 - 2\varepsilon)\, b - \varepsilon\, b \}\, (b + c) = 0,$$

or

$$(3 - \varepsilon)\, b = (\varepsilon - 1)\, (a + c).$$

$$\therefore b = \frac{\varepsilon - 1}{3 - \varepsilon}\, (a + c).$$

Hence, substituting for b in 2,

$$2\, (\varepsilon - 1)\, c = \frac{\varepsilon - 1}{3 - \varepsilon} [\overline{2 - \varepsilon}\ a + \overline{\varepsilon - 1}\ \overline{a + c} + \overline{5 - 3\varepsilon}\ c],$$

or

$$a = 2\, c.$$

Whence

$$b = \frac{\varepsilon - 1}{3 - \varepsilon}\, .3\, c,$$

and substituting in 1, we get

$$l = \frac{\varepsilon - 1}{2}\, \pi\, \frac{\frac{\varepsilon - 1}{3 - \varepsilon} 3 \left[2 + \frac{\varepsilon - 1}{3 - \varepsilon} 3 + 1 \right] c}{\overline{\varepsilon - 1}\, .3 - \overline{2 - \varepsilon}\, \frac{\varepsilon - 1}{3 - \varepsilon} 3},$$

$$= \frac{\varepsilon - 1}{3 - \varepsilon}\, .3\, \pi\, c.$$

Whence

$$\frac{l}{3\, \pi\, \overline{\varepsilon - 1}} = \frac{a}{2\, \overline{3 - \varepsilon}} = \frac{b}{3\, \overline{\varepsilon - 1}} = \frac{c}{3 - \varepsilon}.$$

K is given by

$$\frac{(1 + K)\, \Omega}{\left(1 - \frac{\varepsilon}{2} K\right) H} = \frac{(1 + K)\, \frac{\varepsilon}{2}\, H}{\left(1 - \frac{\varepsilon}{2} K\right) H} = \left[1 \right]$$

$$= \frac{(\varepsilon - 1)\, (6\, \overline{\varepsilon - 1} + 3\, \overline{\varepsilon - 1} + 3 - \varepsilon)}{6\, \overline{\varepsilon - 1}} = \frac{8\varepsilon - 6}{6},$$

$$\therefore \frac{(1 + K)\, \frac{\varepsilon}{2}}{1 + \frac{\varepsilon}{2}} = \frac{8\varepsilon - 6}{8\varepsilon},$$

$$\therefore K = \frac{(4\varepsilon - 3)\left(1 + \frac{\varepsilon}{2}\right)}{2\varepsilon^2} - 1 = \frac{5\varepsilon - 6}{4\varepsilon^2}.$$

We thus see that $l = \pi b$ and $a = 2c$ in general, but the ratio of b to c depends on ε, varying from 0.87 when $\varepsilon = 1.45$ to 1.45 when $\varepsilon = 1.65$.

For transformer iron ε seems, as a rule to lie between 1.5 and 1.6, for the range of the induction over which transformers are worked.

The following table gives the ratio of b to c, and the value of $\frac{\Omega_1}{H_1} c^{4\varepsilon}$ for different values of ε

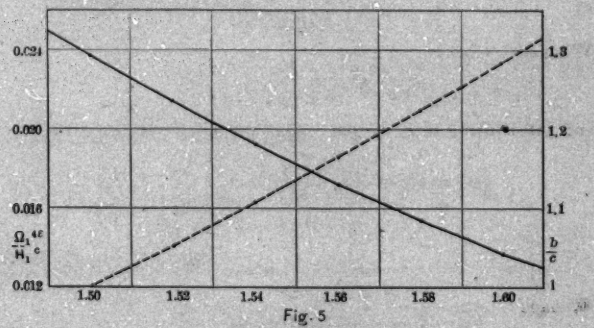

Fig. 5

TABLE I. (FIG. 5).

$\varepsilon =$	1.50	1.52	1.54	1.56	1.58	1.60
$\frac{b}{c} =$	1.000	1.054	1.110	1.167	1.225	1.286
$\frac{\Omega_1}{H_1} c^{4\varepsilon} =$	0.0238	0.0215	0.0193	0.0174	0.0155	0.0138

Where now

$$\Omega_1 = 2 k \rho p \delta^2.$$

$$H_1 = 2 q \eta j f \left\{ \frac{\varphi . V . 10^8 . \sigma}{4 f p q'} \right\}^{\varepsilon} \times 10^{-7}.$$

(§ XVII.)

These, together with the above table, and the relations $l = \pi b$ $a = 2c$, give us the first approximation to the dimensions of the core of the shell transformer which gives minimum losses.

A similar investigation will give us, for a transformer of the core type, (as in § XI).

$$\frac{l}{3\pi(\varepsilon-1)} = \frac{a}{4(3-\varepsilon)} = \frac{b}{3(\varepsilon-1)} = \frac{c}{2(3-\varepsilon)}$$

The following table gives us the ratio of b to c, and the value of $\frac{\Omega_1}{H_1} c^{4\varepsilon}$ for different values of ε, for this case.

TABLE II. (FIG. 6.)

$\varepsilon =$	1.50	1.52	1.54	1.56	1.58	1.60
$\frac{b}{c} =$	0.500	0.527	0.555	0.583	0.613	0.643
$\frac{\Omega_1}{H_1} c^{4\varepsilon} =$	0.0952	0.0884	0.0818	0.0756	0.0694	0.0633

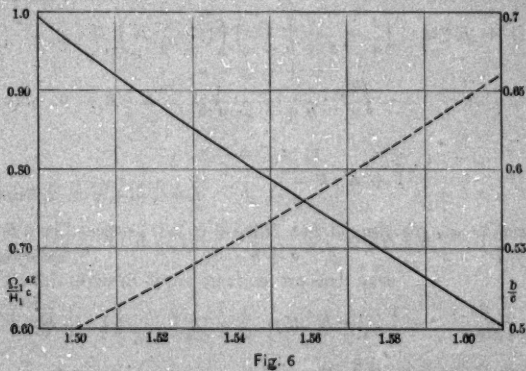

Fig. 6

Where Ω_1 and H_1 have the same values as the preceding case. (See also § XVIII).

If two specimens of transformer iron give about the same hysteresis loss at the same usual induction, (say 3000 lines per sq. cm.), and in one, ε is larger than in the other, it will be found that if transformers be designed, by the above methods to suit the iron, that in which ε is smallest will have least weight. The amounts of copper in the two transformers will not be very different, neither will the waste of energy, but the suitable amount of iron will be smaller when ε is smaller. The radiating surface will also be smaller, so that this transformer would get hotter, if both were used similarly.

APPENDIX C.

Effect of Eddy-Currents.

The equations

$$\frac{\delta W}{\delta l}=0,\quad \frac{\delta W}{\delta a}=0,\quad \frac{\delta W}{\delta b}=0,\quad \frac{\delta W}{\delta c}=0,$$

of Appendix B are,—

$$\frac{2}{2l+\pi\overline{b+c}}(1+K)\,\varrho-\frac{\varepsilon-1}{l}\left(1-\frac{\varepsilon}{2}K\right)H-\frac{1}{l}(1-K)\,E=0, \quad \ldots.1$$

$$\frac{1}{a}(1+K)\,\varrho-\left(\frac{1}{a}-\frac{1}{a+b+c}\right)\left(1-\frac{\varepsilon}{2}K\right)H-\left(\frac{2}{a}-\frac{1}{a+b+c}\right)(1-K)\,E=0, \quad \ldots.2$$

$$\frac{\pi}{2l+\pi\overline{b+c}}(1+K)\,\varrho-\left[\frac{\varepsilon-1}{b}-\frac{1}{a+b+c}\right]\left[1-\frac{\varepsilon}{2}K\right]H-\left[\frac{1}{b}-\frac{1}{a+b+c}\right](1-K)\,E=0, \quad \ldots.3$$

$$\left[\frac{\pi}{2l+\pi\overline{b+c}}+\frac{1}{c}\right](1+K)\,\varrho-\left[\frac{\varepsilon}{c}-\frac{1}{a+b+c}\right]\left[1-\frac{\varepsilon}{2}K\right]H-\left[\frac{2}{c}-\frac{1}{a+b+c}\right](1-K)\,E=0, \quad \ldots.4$$

It is shown, in Appendix B, that if E is zero, these equations have the solution

$$\frac{l_0}{3\pi\overline{\varepsilon-1}}=\frac{a_0}{2\overline{3-\varepsilon}}=\frac{b_0}{3\overline{\varepsilon-1}}=\frac{c_0}{3-\varepsilon},$$

whilst

$$K=\frac{5\varepsilon-6}{4\varepsilon^2},$$

which we will now write K_0; and the absolute values of l_0, a_0, b_0, and c_0, are given by

$$\Omega_1 a_0 c_0 (2 l_0 + \pi \sqrt{b_0 + c_0}) = \frac{\varepsilon}{2} H_1 (a_0 + b_0 + c_0) (a_0 c_0)^{-\varepsilon} (b_0 l_0)^{-\varepsilon+1},$$

or

$$\Omega_0 = \frac{\varepsilon}{2} H_0, \text{ say.}$$

Suppose equations 1, 2, 3, and 4, together with the equation

$$\Omega = \frac{\varepsilon}{2} H + E \qquad \ldots . 5$$

are satisfied by.

$$l = l_0 (1 + \lambda)$$
$$a = a_0 (1 + \alpha)$$
$$b = b_0 (1 + \beta)$$
$$c = c_0 (1 + \gamma)$$

whilst $K = K_0 (1 + \delta K)$.

We will suppose that E is small, and neglect squares of small quantities.

To this order of approximation, we shall have

$$\Omega = \Omega_1 a_0 c_0 (2 l_0 + \pi \sqrt{b_0 + c_0}) \left[1 + \alpha + \gamma + \frac{2 l_0 \lambda + \pi \sqrt{b_0 \beta + c_0 \gamma}}{2 l_0 + \pi \sqrt{b_0 + c_0}}\right]$$

$$= \Omega_0 \left\{1 + \frac{8\varepsilon - 6 . \alpha + 3 \sqrt{\varepsilon - 1}\, \beta + 7 \sqrt{\varepsilon - 3}\, \gamma + 6 \sqrt{\varepsilon - 1}\, \lambda}{8\varepsilon - 6}\right\}$$

or we may write

$$\delta \Omega = \Omega_0 \frac{8\varepsilon - 6\, \alpha + 3 . \sqrt{\varepsilon - 1}\, \beta + 7 \sqrt{\varepsilon - 3}\, \gamma + 6 . \sqrt{\varepsilon - 1}\, \lambda}{8\varepsilon - 6} \qquad (6)$$

again

$$H = H_1 (a_0 + b_0 + c_0) (a_0 c_0)^{-\varepsilon} (b_0 l_0)^{-\varepsilon+1}$$

$$\left\{ 1 - \varepsilon . \overline{\alpha + \gamma} - \overline{\varepsilon - 1} . \overline{\beta + \lambda} + \frac{a_0 \alpha + b_0 \beta + c_0 \gamma}{a_0 + b_0 + c_0} \right\}$$

$$= H_0 \left\{ 1 - \frac{\overline{8\varepsilon - 6}\,\alpha + 3 . \overline{\varepsilon - 1}\,\beta + \overline{7\varepsilon - 3}\,\gamma + 6 . \overline{\varepsilon - 1}\,\lambda}{6} \right\}$$

thus

$$\delta H = - H_0 \frac{\overline{8\varepsilon - 6}\,\alpha + 3 . \overline{\varepsilon - 1}\,\beta + \overline{7\varepsilon - 3}\,\gamma + 6 . \overline{\varepsilon - 1}\,\lambda}{6}$$

$$= - Q_0 \frac{\overline{8\varepsilon - 6}\,\alpha + 3 . \overline{\varepsilon - 1}\,\beta + \overline{7\varepsilon - 3}\,\gamma + 6 . \overline{\varepsilon - 1}\,\lambda}{3\varepsilon}$$

$$= - \frac{8\varepsilon - 6}{3\varepsilon} \delta Q, \qquad (7)$$

again

$$\left.\begin{aligned} 1 + K_0 &= \frac{4\varepsilon^2 + 5\varepsilon - 6}{4\varepsilon^2} = \frac{(\varepsilon + 2)(4\varepsilon - 3)}{4\varepsilon^2} \\ 1 - \frac{\varepsilon}{2} K_0 &= \frac{8\varepsilon - 5\varepsilon + 6}{8\varepsilon} = \frac{3(\varepsilon + 2)}{8\varepsilon} \\ 1 - K_0 &= \frac{4\varepsilon^2 - 5\varepsilon + 6}{4\varepsilon^2} \end{aligned}\right\} \qquad (8)$$

Putting

$l = l_0 (1+\lambda)$, $a = a_0 (1+\alpha)$, $b = b_0 (1+\beta)$, $c = c_0 (1+\gamma)$,

and

$$K = K_0 (1 + \delta K),$$

in equation 1, it becomes, by virtue of eqns. 6, 7, & 8,

$$\frac{2}{\pi (8\varepsilon - 6)} Q_0 K_0 \delta K + \frac{\varepsilon - 1}{3\pi (\varepsilon - 1)} Q_0 K_0 \delta K$$

$$- \frac{2}{(8\varepsilon - 6)\pi} \frac{6 . \overline{\varepsilon - 1}\,\lambda + 3\,\overline{\varepsilon - 1}\,\beta + \overline{3 - \varepsilon}\,\gamma}{8\varepsilon - 6} \frac{(\varepsilon + 2)(4\varepsilon - 3)}{4\varepsilon^2} Q_0$$

$$+ \frac{\varepsilon - 1}{3\pi (\varepsilon - 1)} Q_0 \frac{2}{\varepsilon} . \frac{3(\varepsilon + 2)}{8\varepsilon} \lambda$$

$$+\left\{\frac{2}{\pi\,(8\,\varepsilon-6)}\cdot\frac{(\varepsilon+2)\,(4\,\varepsilon-3)}{4\,\varepsilon^2}+\frac{\varepsilon-1}{3\,\pi\,(\varepsilon-1)}\cdot\frac{3(\varepsilon+2)}{8\,\varepsilon}\cdot\frac{8\,\varepsilon-6}{3\,\varepsilon}\right\}$$

$$\frac{\sqrt{8\,\varepsilon-6}\;\;\alpha+3.\sqrt{\varepsilon-1}\;\;\beta+7\sqrt{\varepsilon-3}\;\gamma+6\sqrt{\varepsilon-1}\;\lambda}{8\,\varepsilon-6}\,\mathcal{Q}_0$$

$$-\frac{1}{\pi\,(8\,\varepsilon-6)}\cdot\frac{4\,\varepsilon^2-5\,\varepsilon+6}{4\,\varepsilon^2}\,E=0,$$

Reducing to

$$\frac{4}{3}\,\varepsilon\;\alpha+\frac{1}{2}\sqrt{\varepsilon-1}\;\beta+\frac{7\,\varepsilon+3}{6}\,\gamma+\varepsilon\,\lambda$$

$$=\frac{4\,\varepsilon^2-5\,\varepsilon+6}{3\,(\varepsilon-1)\,(\varepsilon+2)}\frac{E}{\mathcal{Q}_0}-\frac{8\,\varepsilon\,(5\,\varepsilon-6)}{3\,(8\,\varepsilon-6)\,(\varepsilon+2)}\,\delta\;K. \quad \ldots.9$$

Similarly equation 2 becomes

$$\frac{1}{2\,(3-\varepsilon)}\,\mathcal{Q}_0\,K_0\,\delta\;K+\left[\frac{\varepsilon}{2\,(3-\varepsilon)}-\frac{1}{6}\right]\mathcal{Q}_0\,K_0\,\delta\;K$$

$$-\frac{1}{2(3-\varepsilon)}\,\mathcal{Q}_0\,\frac{(\varepsilon+2)\,(4\,\varepsilon-3)}{4\,\varepsilon^2}\,\alpha$$

$$+\left\{\frac{\varepsilon\,\alpha}{2\,(3-\varepsilon)}-\frac{1}{6}\,\frac{2\sqrt{3-\varepsilon}\;\alpha+3\sqrt{\varepsilon-1}\;\beta+3\sqrt{3-\varepsilon}\;\gamma}{6}\right\}\frac{3\,(\varepsilon+2)}{8\,\varepsilon}\cdot\frac{2}{\varepsilon}\,\mathcal{Q}_0$$

$$+\left\{\frac{1}{2\,(3-\varepsilon)}\cdot\frac{(\varepsilon+2)\,(4\,\varepsilon-3)}{4\,\varepsilon^2}+\left[\frac{\varepsilon}{2\,(3-\varepsilon)}-\frac{1}{6}\right]\frac{3(\varepsilon+2)}{8\,\varepsilon}\,\frac{8\,\varepsilon-6}{3\,\varepsilon}\right\}$$

$$\left\{\frac{\sqrt{8\,\varepsilon-6}\;\;\alpha+3.\sqrt{\varepsilon-1}\;\beta+7\sqrt{\varepsilon-3}\;\gamma+6\sqrt{\varepsilon-1}\,\lambda}{8\,\varepsilon-6}\right\}\mathcal{Q}_0$$

$$-\left\{\frac{2}{2\,(3-\varepsilon)}-\frac{1}{6}\right\}\frac{4\,\varepsilon^2-5\,\varepsilon+6}{4\,\varepsilon^2}\,E=0,$$

which reduces to

$$\frac{5\,\varepsilon-3}{2}\,\varepsilon\;\alpha+\frac{5\,\varepsilon-3}{4}\,(\varepsilon-1)\;\beta+\frac{9\,\varepsilon^2-2\,\varepsilon-3}{4}\,\gamma+2\,\varepsilon\,(\varepsilon-1)\,\lambda$$

$$=\frac{3+\varepsilon}{6\,(\varepsilon+2)}\,(4\,\varepsilon^2-5\,\varepsilon+6)\,\frac{E}{\mathcal{Q}_0}-\frac{2\,\varepsilon\,(5\,\varepsilon-6)}{3\,(\varepsilon+2)}\,\delta\;K. \quad \ldots.10$$

Equation 3 becomes

$$\frac{1}{8\varepsilon-6}\,\mathcal{Q}_0\,K_0\,\delta\,K+\left[\frac{\varepsilon-1}{3\overline{\varepsilon-1}}-\frac{1}{6}\right]\mathcal{Q}_0\,K_0\,\delta\,K$$

$$-\frac{1}{8\varepsilon-6}\cdot\frac{(6.\overline{\varepsilon-1}\,\lambda+3\,\overline{\varepsilon-1}\,\beta+3-\overline{\varepsilon}\,\gamma)}{8\varepsilon-6}\cdot\frac{(4\varepsilon-3)(\varepsilon+2)}{4\varepsilon^2}\,\mathcal{Q}_0$$

$$+\left\{\frac{\varepsilon-1}{3(\varepsilon-1)}\,\beta-\frac{1}{6}\frac{2.3-\varepsilon\,\alpha+3.\overline{\varepsilon-1}\,\beta+3-\overline{\varepsilon}\,\gamma}{6}\right\}\frac{3.\overline{\varepsilon+2}}{8\varepsilon}\cdot\frac{2}{\varepsilon}\,\mathcal{Q}_0$$

$$+\left\{\frac{1}{8\varepsilon-6}\cdot\frac{(4\varepsilon-3)(\varepsilon+2)}{4\varepsilon^2}+\left[\frac{\varepsilon-1}{3\overline{\varepsilon-1}}-\frac{1}{6}\right]\frac{3.\overline{\varepsilon+2}}{8\varepsilon}\cdot\frac{8\varepsilon-6}{3\varepsilon}\right\}$$

$$\left\{\frac{8\,\overline{\varepsilon-6}\,\alpha+3.\,\overline{\varepsilon-1}\,\beta+7\,\overline{\varepsilon-3}\,\gamma+6\,\overline{\varepsilon-1}\,\lambda}{8\varepsilon-6}\right\}\mathcal{Q}_0$$

$$-\left\{\frac{1}{3(\varepsilon-1)}-\frac{1}{6}\right\}\frac{4\varepsilon^2-5\varepsilon+6}{4\varepsilon^2}\,E=0,$$

which reduces to

$$\frac{5\varepsilon-3}{6}\,\alpha+\beta+\frac{2}{3}\varepsilon\gamma+\frac{\varepsilon-1}{2}\,\lambda$$

$$=\frac{1}{6}\frac{3-\varepsilon}{(\varepsilon-1)(\varepsilon+2)}(4\varepsilon^2-5\varepsilon+6)\frac{E}{\mathcal{Q}_0}-\frac{2\varepsilon(5\varepsilon-6)}{3(4\varepsilon-3)(\varepsilon+2)}\,\delta\,K \qquad \ldots\ldots 11$$

Equation 4 becomes

$$\left[\frac{1}{8\varepsilon-6}+\frac{1}{3-\varepsilon}\right]\mathcal{Q}_0\,K_0\,\delta\,K+\left[\frac{\varepsilon}{3-\varepsilon}-\frac{1}{6}\right]\mathcal{Q}_0\,K_0\,\delta\,K$$

$$-\left\{\frac{1}{8\varepsilon-6}\,\frac{6.\,\overline{\varepsilon-1}\,\lambda+3.\,\overline{\varepsilon-1}\,\beta+3-\overline{\varepsilon}\,\gamma}{8\varepsilon-6}\right.$$

$$\left.+\frac{1}{3-\varepsilon}\,\gamma\right\}\left\{\frac{(4\varepsilon-3)(\varepsilon+2)}{4\varepsilon^2}\,\mathcal{Q}\right.$$

$$+\left\{\frac{\varepsilon}{3-\varepsilon}\,\gamma-\frac{1}{6}\frac{2.3-\overline{\varepsilon}\,\alpha+3.\overline{\varepsilon-1}\,\beta+3-\overline{\varepsilon}\,\gamma}{6}\right\}\frac{3.\overline{\varepsilon+2}}{8\varepsilon}\cdot\frac{2}{\varepsilon}\,\mathcal{Q}_0$$

$$+\left\{\left[\frac{1}{8\,\varepsilon-6}+\frac{1}{3-\varepsilon}\right]\frac{(4\,\varepsilon-3)(\varepsilon+2)}{4\varepsilon^2}+\left[\frac{\varepsilon}{3-\varepsilon}-\frac{1}{6}\right]\frac{3.\overline{\varepsilon+2}}{8\,\varepsilon}\cdot\frac{8\varepsilon-6}{3\,\varepsilon}\right\}$$

$$\left\{\frac{\overline{8\,\varepsilon-6}\;\alpha+3.\overline{\varepsilon-1}\;\beta+\overline{7\,\varepsilon-3}\;\gamma+6.\overline{\varepsilon-1}\;\lambda}{8\,\varepsilon-6}\right\}\mathcal{Q}_0$$

$$-\left[\frac{2}{3-\varepsilon}-\frac{1}{6}\right]\frac{4\,\varepsilon-5\,\varepsilon+6}{4\,\varepsilon^2}\,E=0,$$

which reduces to

$$(9\;\varepsilon^2-2\,\varepsilon-3)\;\alpha+4\,\varepsilon\;(\varepsilon-1)\;\beta+2\,(4\,\varepsilon^2-\varepsilon+3)\,\gamma+(\varepsilon-1)\,(7\,\varepsilon+3)\,\lambda$$

$$=\frac{9+\varepsilon}{2\,(\varepsilon+2)}\,(4\,\varepsilon^2-5\,\varepsilon+6)\,\frac{E}{\mathcal{Q}_0}-\frac{4}{3}\,\frac{\varepsilon.(7\,\varepsilon-3)(5\,\varepsilon-6)}{(4\,\varepsilon-3)\,(\varepsilon+2)}\,\delta\,K \quad \ldots.12$$

Whilst equation 5, *i. e.*,

$$\mathcal{Q}=\frac{\varepsilon}{2}\,H+E,$$

becomes

$$\delta\,\mathcal{Q}=\frac{\varepsilon}{2}\,\delta\,H+E,$$

or

$$\delta\,\mathcal{Q}\left[1+\frac{8\,\varepsilon-6}{3\,\varepsilon}\cdot\frac{\varepsilon}{2}\right]=E,$$

or

$$\overline{8\;\varepsilon-6}.\alpha+3.\overline{\varepsilon-1}\;\beta+\overline{7\,\varepsilon-3}\;\gamma+6.\overline{\varepsilon-1}\;\lambda=\frac{3\,(4\,\varepsilon-3)}{2\,\varepsilon}\,\frac{E}{\mathcal{Q}_0} \quad \ldots.13$$

If we put $\frac{E}{\mathcal{Q}_0}=S$, for brevity, we may write equations 9, 10, 11, 12 and 13, respectively as follows:

$$\tfrac{2}{3}\,(4\,\varepsilon-3)\,\varepsilon\,\alpha+\tfrac{1}{4}\,(4\,\varepsilon-3)\,(\varepsilon-1)\,\beta+\tfrac{1}{12}\,(4\,\varepsilon-3)\,(7\,\varepsilon+3)\,\gamma$$

$$+\tfrac{1}{3}\,(4\,\varepsilon-3)\,\varepsilon\,\lambda,$$

$$=\frac{(4\,\varepsilon-3)\,(4\,\varepsilon^2-5\,\varepsilon+6)}{6\,(\varepsilon-1)\,(\varepsilon+2)}\,S-\tfrac{2}{3}\,\frac{\varepsilon\,(5\,\varepsilon-6)}{\varepsilon+2}\,\delta\,K, \quad \ldots.14$$

$$\tfrac{1}{2}\,(5\,\varepsilon-3)\,\varepsilon\,\alpha+\tfrac{1}{4}\,(5\,\varepsilon-3)\,(\varepsilon-1)\,\beta+\tfrac{1}{4}\,(9\,\varepsilon^2-2\,\varepsilon-3)\gamma$$

$$+2\,\varepsilon\,(\varepsilon-1)\,\lambda$$

$$=\frac{(\varepsilon+3)\,(4\,\varepsilon^2-5\,\varepsilon+6)}{6\,(\varepsilon+2)}\,S-\tfrac{2}{3}\,\frac{\varepsilon\,(5\,\varepsilon-6)}{\varepsilon+2}\,\delta\,K, \quad \ldots.(15)$$

$$\tfrac{1}{3}(4\varepsilon-3)(5\varepsilon-3)\alpha+(4\varepsilon-3)\beta+\tfrac{2}{3}(4\varepsilon-3)\varepsilon\gamma\cdot$$
$$+\tfrac{1}{3}(4\varepsilon-3)(\varepsilon-1)\lambda$$
$$=\frac{1}{6}\frac{(4\varepsilon-3)(3-\varepsilon)(4\varepsilon^2-5\varepsilon+6)}{(\varepsilon-1)(\varepsilon+2)}S-\frac{2}{3}\frac{\varepsilon(5\varepsilon-6)}{\varepsilon+2}\delta K, \quad \ldots.(16)$$

$$\frac{1}{2}\frac{4\varepsilon-3}{7\varepsilon-3}(9\varepsilon^2-2\varepsilon-3)\alpha+2\frac{4\varepsilon-3}{7\varepsilon-3}\cdot\varepsilon(\varepsilon-1)\beta$$
$$+\frac{4\varepsilon-3}{7\varepsilon-3}(4\varepsilon^2-\varepsilon+3)\gamma+\frac{1}{2}\cdot\frac{4\varepsilon-3}{7\varepsilon-3}(7\varepsilon+3)(\varepsilon-1)\lambda$$
$$=\frac{(4\varepsilon-3)(\varepsilon+9)(4\varepsilon^2-5\varepsilon+6)}{6(\varepsilon+2)(7\varepsilon-3)}S-\frac{2}{3}\frac{\varepsilon(5\varepsilon-6)}{\varepsilon+2}\delta K, \quad \ldots.(17)$$

$$2(4\varepsilon-3)\alpha+3(\varepsilon-1)\beta+(7\varepsilon-3)\gamma+6(\varepsilon-1)\lambda$$
$$=\frac{3}{2}\frac{4\varepsilon-3}{\varepsilon}S. \quad \ldots.(18)$$

Subtracting eqn. 14 from 15 we get,—

$$\frac{3-\varepsilon}{6}\alpha+\frac{\varepsilon-1}{4}\beta+\frac{3-\varepsilon}{12}\gamma-\frac{1}{2}\lambda$$
$$+\frac{(2-\varepsilon)(4\varepsilon^2-5\varepsilon+6)}{6(\varepsilon-1)(\varepsilon+2)}S=0. \quad \ldots.(19)$$

Subtracting eqn. 16 from 14 we get,—

$$\frac{3-\varepsilon}{6}\alpha-\frac{5-\varepsilon}{4}\beta+\frac{3-\varepsilon}{12}\gamma+\frac{1}{2}\lambda$$
$$+\frac{(2-\varepsilon)(4\varepsilon^2-5\varepsilon+6)}{6(\varepsilon-1)(\varepsilon+2)}S=0. \quad \ldots.(20)$$

Now subtracting eqn. 20 from 19 we get,—

$$\beta=\lambda.$$

Thus equations 19 and 20 are equivalent to,—

$$2\alpha+\gamma-3\lambda+\frac{2(2-\varepsilon)(4\varepsilon^2-5\varepsilon+6)}{(3-\varepsilon)(\varepsilon-1)(\varepsilon+2)}S=0. \quad \ldots.(21)$$

Again, subtracting eqn. 16 from 17, we get,—

$$\tfrac{1}{3}(4\varepsilon-3)(3-\varepsilon)\alpha+(2\varepsilon^2-9\varepsilon+3)\beta+\tfrac{1}{3}(3-\varepsilon)(2\varepsilon+3)\gamma+3(\varepsilon-1)\lambda$$

$$+\frac{4\,\varepsilon\,(2-\varepsilon)\,(4\,\varepsilon^2-5\,\varepsilon+6)}{3\,(\varepsilon-1)\,(\varepsilon+2)}\,S=0,$$

or since $\beta=\lambda$

$$\tfrac{1}{3}\,(4\,\varepsilon-3)\,\alpha+\tfrac{1}{3}\,(2\,\varepsilon+3)\,\gamma-2\,\varepsilon\,\lambda$$

$$+\frac{4\,\varepsilon\,(2-\varepsilon)\,(4\,\varepsilon^2-5\,\varepsilon+6)}{3\,(\varepsilon-1)\,(\varepsilon+2)\,(3-\varepsilon)}\,S=0. \quad \ldots.(22)$$

Eliminating λ between eqns 21 and 22 we get,—

$$3\,\alpha-3\,\gamma=0,$$

or

$$\alpha=\gamma.$$

Hence, from equations 21 or 22,—

$$\alpha-\lambda+\frac{2}{3}\,\frac{(2-\varepsilon)\,(4\,\varepsilon^2-5\,\varepsilon+6)}{(3-\varepsilon)\,(\varepsilon-1)\,(\varepsilon+2)}\,S=0, \quad \ldots. \quad (23)$$

and equation 18 becomes

$$(5\,\varepsilon-3)\,\alpha+3\,(\varepsilon-1)\,\lambda-\frac{1}{2}\,\frac{4\,\varepsilon-3}{\varepsilon}\,S=0, \quad \ldots.(24)$$

whence

$$\left.\begin{aligned}\alpha=\gamma&=-\left\{\frac{(2-\varepsilon)(4\varepsilon^2-5\varepsilon+6)}{(3-\varepsilon)(4\varepsilon-3)(\varepsilon+2)}-\frac{1}{4\,\varepsilon}\right\}S,\\ \beta=\lambda&=\left\{\frac{1}{3}\,\frac{(2-\varepsilon)(4\varepsilon^2-5\varepsilon+6)}{(3-\varepsilon)(4\varepsilon-3)(\varepsilon+2)}\cdot\frac{5\varepsilon-3}{\varepsilon-1}+\frac{1}{4\,\varepsilon}\right\}S.\end{aligned}\right\} \quad (25)$$

Thus if $\varepsilon=1.6$ we have,—

$$\alpha=\gamma=-0.036\,\frac{E}{\Omega_0}, \qquad \text{and } \beta=\lambda=0.691\,\frac{E}{\Omega_0},$$

whilst if $\varepsilon=1.5$ we get,—

$$\alpha=\gamma=-0.071\,\frac{E}{\Omega_0}, \qquad \text{and } \beta=\lambda=0.881\,\frac{E}{\Omega_0}.$$

The values of α, β, γ, and λ, for other values of ε will be given with sufficient accuracy, by interpolation or extrapolation; for although α, β, γ, and λ, change considerably with change of ε, this change has but a small effect on the dimensions, when, as we have assumed in the calculations, the corrections to be made are small.

APPENDIX D.

Proof that the Transformers designed have Minimum Losses.

In §§ x and xi and in Appendices B and C we have shown how to find the proportions and size of the transformer having minimum waste of energy consistent with certain external restrictions. The method of procedure was to equate to zero the first differentials, with respect to the dimensions, of the expression for the energy wasted. This method, however, would equally well find us the dimensions, if such exist, which will make the energy wasted a maximum, and, for the sake of completeness, it is necessary to prove that the solution given corresponds to an absolutely minimum waste, that is, that no change in *any* of the dimensions will diminish the waste of energy.

In the first place, we may, for this purpose, neglect the effect of the eddy-currents, for, although this may somewhat change the dimensions corresponding to the stationary point, it will certainly not change a minimum point to a maximum or *vice versa*.

Using the same notation as in Appendix B the problem is to find the minimum value of $H + \mathcal{Q}$, when $\mathcal{Q}$ and H are connected by the relation

$$\mathcal{Q} = \frac{\epsilon}{2} H$$

This is the same as finding the minimum value of

$$\mathcal{Q}\left[1 + K\right] + H\left[1 - \frac{\epsilon}{2} K\right]$$

subject to the condition

$$\mathcal{Q} = \frac{\epsilon}{2} H$$

where K is any constant. If K has the value $\frac{5\,\epsilon - 6}{4\,\epsilon^2}$, then, the unique solution we obtain for the stationary value of

$$\mathcal{Q}\left[1 + K\right] + H\left[1 - \frac{\epsilon}{2} K\right],$$

satisfies

$$\mathcal{Q} = \frac{\epsilon}{2} H$$

naturally.

Consider now the problem of finding the minimum value of

$$\mathcal{Q}\left[1+K\right]+H\left[1-\frac{\varepsilon}{2}K\right]$$

where

$$K=\frac{5\varepsilon-6}{4\varepsilon^2},$$

and $\mathcal{Q}$ and H are unconnected by any relation. We are able to find a single possible stationary value for the function, and the only question is whether this value is an absolute minimum or not. Since the variables are independent, we may consider the effects of their variations separately.

The function

$$X=\mathcal{Q}\left[1+K\right]+H\left[1-\frac{\varepsilon}{2}K\right]$$

is continuous in all the variables. Let us consider the effect of the variation of a, supposing b, c, and l, to be fixed.

When a is large, $\mathcal{Q}$ is large, and therefore X is large.

When a is small, H is large, and therefore X is large.

When a is of finite size, accordingly, X is smaller than when a is very large or very small, and it therefore passes through a minimum for some finite value of a, since it is continuous.

Similar argument precisely will apply to b, c, and l, and accordingly the unique solution obtained for the stationary value for X in Appendix B, must correspond to an absolute minimum, when $\mathcal{Q}$ and H are not connected by any relation.

This solution is the same as that obtained for our problem of finding the minimum value of $H+\mathcal{Q}$ when

$$\mathcal{Q}=\frac{\varepsilon}{2}H,$$

that is, it gives us the stationary value of $H+\mathcal{Q}$ consistent with the above condition.

Again, since, at the point obtained in this solution,

$$X\left[\equiv\mathcal{Q}\sqrt{1+K}+H\sqrt{1-\frac{\varepsilon}{2}K}\right]$$

increases with all changes in the variables it necessarily increases with those particular changes which are consistent with the condition

$$\mathcal{Q}=\frac{\varepsilon}{2}H$$

that is, the solution given makes

$$\mathcal{Q}\left[1+K\right]+H\left[1-\frac{\varepsilon}{2}K\right]$$

an absolute minimum, when the variables are connected by the relation

$$Q = \frac{\epsilon}{2} H.$$

Accordingly the solution found makes $Q + H$ an absolute minimum subject to the condition

$$Q = \frac{\epsilon}{2} H.$$

Discussion.

Dr. M. I. Pupin:—Mr. President, I do not think that there is much to discuss in connection with this paper. There is so much difference of opinion, and there naturally must be a great deal of difference of opinion in things depending on individual taste. The old Latin proverb, "*De gustibus non est disputandum*," applies eminently well to the design of any apparatus. The designing of machinery is a work of art, as well as of pure reasoning. A man may master the science of electricity and yet not be able to design a completely satisfactory piece of apparatus, because he lacks the artistic element. It is a thing that cannot be taught. It is just like the saying of the Greek philosopher Socrates, that you cannot teach a man virtue. It is a thing that has to be born in a man, and then developed by experience. The art of designing machinery has to be learned by practical experience, and a man will profit by his practical experience, and become a master of designing only if he has a taste for it. The experience will develop that which is in him. If he has it not in him, all the teaching in the world, and all the experience will not carry him very far. For that reason, I think, instructors in colleges do not insist very much upon the theory of design, because there is no clear cut theory relating to it. You have a theory of electricity. There are the fundamental principles and their generalizations, and you have the materials, and a man has to know the properties of materials and to combine his knowledge of electricity with his knowledge of materials to produce a method of design, and each man will have a different method; his own method of designing, but of course, all these different methods will lead to the same result. You see in this paper a calculation of what should be the dimensions of a transformer that should have the least loss. Taking a Westinghouse transformer as an example, it is found that this transformer has much more iron than this paper recommends. In another case, that of the Mordey transformer, a different relation is found. Again, the author finds that the thickness of the iron plates should be $7\frac{1}{2}$ mils, and Prof. Bedell finds that it should be 16 mils. I think the American practice is to use 12 mils. So who is right? Probably they are all right, for each one of them had different conditions to fulfil, and it is impossible to design a transformer which will satisfy all conditions. On the whole I do not think that this paper will lead to anything that could be used as a standard for all cases. But at the same time it is a very good thing to have in a paper like this, filed away in our Transactions, and ready for reference, the various steps, the essential steps of reasoning that a man has to go through in designing a transformer. We certainly who teach in colleges are very glad to have such a reference, because it saves us a great deal of tedious discussion. A student comes to us and says: "What am I going

to do about this transformer design? What is the next step to take?" The teacher, you know, in a great many cases is not quite sure what the next step is, and he does not care to display his ignorance before the student. So he feels very much like saying to the eager student: "you will have to look that up in the electrical literature." I must say that so far, in electrical literature, there never was a convenient reference of this kind. The college professors of limited experience in transformer design (and their number is very large) can now say to students: "Read Mr. Carter's paper; it will give you all the essential points in transformer design." It takes a great deal of unnecessary, disagreeable work out of our hands; and it is disagreeable, to be sure, because not every teacher can be expected to know all the details of electrical apparatus design. Unless a man has been employed in some factory and made his living by designing, he cannot be expected to know much about it. As a mere scientific study the thing is dreadfully uninteresting. But it is a necessary evil, and very necessary indeed. In fact the success, sometimes of a company, depends on the appropriate design of its machinery. I was told some time ago that a man who was connected for a long time with a large electrical manufacturing company, stood very high with the company because he was a most eminent designer. He could design better than anybody else because he was a born artist in matters of design. That man could tell you more about a machine by merely looking at it than an ordinary electrician could by elaborate tests. The success of the company in a great measure depended on him. That shows how necessary a thing it is—this work of designing which to many of us appears dry. We ought to be very grateful to Mr. Carter for having taken the trouble to put down in this admirable, systematic way, the method of designing a transformer; and I only wish that somebody else who has had a large practical experience would come up and tell us all about designing an alternator, synchronous motors, induction motors and all that sort of thing. We would then be in a very fair position to train our young men and to develop their latent faculty of designing.

Prof. W. S. Franklin:—I wish to express my concurrence with the views of Dr. Pupin in the matter of teaching transformer design to students. The "cut and try" method is no doubt the method of practice, but it is not a method which lends itself with any convenience whatever to teaching. We have not had heretofore a mathematical statement of the principles of proportioning transformer parts, or in short we have had no method of design. We have known indeed precisely how transformers act, we have known how to calculate the secondary turns when the primary turns are given, we have known how to calculate losses, regulation, and every item of their behavior, but these are only the foundations of designing, and it seems to me that Mr. Carter's paper is the first that has been given on transformer design.

I may be permitted to say further that a consideration of limiting cost might easily be formulated and used in connection with Mr. Carter's method for determining rationally the absolute size of a transformer, instead of the choice of an arbitrary current density in the coils. This formulation is outlined as follows:

The cost, \$, is approximately:

$$\$ = ac + bi$$

where c is the weight of copper, i the weight of iron, and a and b are constants. The interest charge is proportional to \$ and may therefere be written

$$\% = a'c + b'i.$$

Take, now. Mr. Carter's expression for total loss, W, in terms of the yet undetermined dimensions of the transformer. The yearly value, Y, of this loss is at once known in the terms of the value of power, and the yearly total cost of the transformer is $Y + \%$ which is completely expressible, as above stated, in terms of the various items of the yet undetermined design, and these various items may then be determined so as to make the function $Y + \%$ a minimum. In this way the design would be determined without any arbitrary basis whatever. This method would always give a cheaper transformer than the transformer of maximum efficiency.

[Mr. W. B. Jackson's remarks delayed for revision.—EDITOR.]

MR. TOWNSEND WOLCOTT:—The question as to whether transformers in use to-day are designed by "rule of thumb" or not seems to be still open. I know within the last year or two I have seen several statements, especially from English electricians; —(I think one was Mr. Swinburne, another was Prof. Fleming) —that owing to the fact that there is no simple mathematical expression, in fact even a complicated one, for the relation of magnetism in iron to the exciting current, it is impossible to design a transformer entirely on a scientific basis; the design is practically "rule of thumb." On the other hand, some other electricians, more particularly Prof. Carhart, last year said that he gets practically accurate results by assuming the sine form of wave of magnetism. In the first place, he may not have exactly a sine current, but assumes a sine form for both the current wave and also the wave of magnetism, and he says he gets results that are as accurate as you get in direct current machinery. It would seem that the "rule of thumb" practice is going out of designing at all events, if it is not already out.

PROF. FRANKLIN:—Mr. President, if I may impose upon the good nature of the INSTITUTE yet a few moments more, I may be allowed perhaps to say a word on that matter, as to what the design of transformers consists of at the present time. It seems to me—of course, I don't know very much about it, I will admit that—but it seems to me that the thing we know about transformers and the thing which practical men know about transformers is *how transformers act*. After a transformer is made,

after you have decided upon the items in its design, you can calculate the ratio of electromotive force, you can calculate the magnetizing current, you can calculate the hysteresis and eddy current losses and the ohmic losses and all the details, without exception. That of course constitutes, as matter of fact, the basis of the paper to-day. Mr. Carter takes all that for granted. But the designing of a transformer is a matter of correlating these various items, and I think there can be no question but that the design heretofore has been a "cut and try" method of correlation and not a rational method of correlation. Of course it is not a matter of a single designer's beginning with Adam and working his own "cut and try" himself. He makes much of the practice as it exists at the present day, which represents the results of all the cutting and trying which has been done before him. A designer at the present time perhaps loses sight of the fact that the real essence of his design is "cut and try," because he does not carry out all of the cutting and trying himself and he is inclined to think that his knowledge of the action of the transformer is what he uses in designing. I think not. I think that he uses this merely in calculating what the transformer will do after he has designed it. Then he will design another with say two or three more turns of copper, and he will calculate what that will do. Then he will design another with two or three more sheets of iron and he will calculate what that will do. If that is not cutting and trying I don't know what cutting and trying is. Of course I may be mistaken in supposing that practical men design transformers in that way. I have designed a good many transformers, but they never were built, you know, so I cannot say anything from a practical point of view.

I cannot withstand the desire, Mr. President, to say one thing more, not in direct connection with the subject in hand, but in connection with the remark which was made by Mr. Jackson. He said that college men do not teach the actual practical methods of engineering, and the actual practical significance of the various points they take up. Now I think that as a rule no class of men understand better the worthlessness of what they teach than college men themselves. I think they appreciate the truth of a remark, if I may be allowed to quote German—a remark which occurs in Goethe's Faust, where Mephistophiles turning gravely on a student who has just come to confer with the professor: "Gray, my dear friend, is all theory, and green only is the tree of life."

DR. PUPIN:—If I may be permitted just to say one word more, I would like to supplement the quotation that Professor Franklin has made with reference to the teaching of design. Faust also says:

"So muss ich dem mit saurem Schweiss
Euch lehrn, was ich selbst nicht weiss,"

which I can translate for you in English, but of course not so as

to produce Goethe's classical rhythm—"And so with the sweat of my brow I must teach you what I do not understand myself."

As far as I can see this paper is just as practical as it can possibly be. It is limited to the design of a transformer under perfectly specified conditions. It does not speak of the oil transformer, because that is a separate matter. I do not see why the question of enclosing the transformer in oil is any more practical than the subject that this writer has discussed. Everything is practical, but of course we cannot take more than a certain number of steps at a time, and after we have taken those steps securely and safely we can take the next step. The question of whether we enclose the transformer in oil or not, is no more practical than giving the transformer the proper dimensions. A previous speaker admits that we consider also when we take up this question of oil: will not oil dropping from transformers hung upon poles on the clothes of people bring the company into difficulties? That is a very practical question, but it is hardly within the limits of design. It brings in social questions and legal questions. In this connection it might be well to repeat a story told concerning a strange experience with the experiment in conveying power from Lauffen to Frankfort. The original design was all right, that is, the engineers thought it was all right, but they found it was all wrong, the thing would not work; and when they came to examine it they found that they made a mistake in making the insulators white, because the little boys in the fields had their attention attracted to those white insulators and thought they were good marks to throw stones at, so they threw stones at them and broke them, and that affected the whole plant and caused a tremendous leakage. The insulators were painted gray and the trouble was over. Well, that is hardly within the limits of designing conductors to transmit power, and yet it was of vital importance. At some date it may be of very great importance to know whether we should paint our transformers hung up on a pole, green or red or gray- but if we do, it will be owing to some psychological or perhaps political reason. All questions are practical, but of course certain questions are more directly within the limits of transformer design than others, and I would not blame Mr. Carter for not spreading himself too wide. I think he has spread himself wide enough as it is.

Mr. Walter S. Moody:—As one who is actively engaged in every day practical design, I would like to make a few remarks.

First, I can assure these gentlemen who question the amount of use that is made of theory in every day design of electrical apparatus, that as far as my own work is concerned all the theory that college men develop for us finds ample use in every day practice.

There is no class of electrical apparatus designed to-day, I believe, by the best companies, whose proportions are not just as carefully analyzed as Mr. Carter's paper analyzes transformer

proportions. But, of course, there are a great number of practical difficulties and considerations that modify the purely theoretical formula, or, more correctly speaking, our theoretical formula cannot be broad and complicated enough to include all the actual conditions presented. Mr. Carter's paper, for instance, while very complete on the points considered, leaves entirely out of consideration such important facts as might entirely overpower the importance of the features which he discusses. Perhaps the most important factor which Mr. Carter does not in any way refer to is the question of heating, and the necessary radiating surfaces to dispose of the losses within the transformer. It would be perfectly possible to apply the formulæ developed by Mr. Carter, and which in themselves are undoubtedly perfectly correct, and yet obtain a design entirely impractical because of the lack of the necessary radiating surface to dispose of the heat.

Again, Mr. Carter's calculations all contain a constant which he designates as "p," and which expresses in per cent. the relation between the cross-section between the primary and secondary copper and the area of the wire space. In the calculations this figure is assumed, or rather it seems to be based on the actual per cent. found to express this relation in the transformers under consideration. Of course, in actual design this question has to be given careful study in order to make the percentage as high as possible, and still get the necessary insulation and the best distribution of it.

What I would like to bring out is that there is really great latitude in the choice of the best proportions for transformer design. One who has carefully figured the proportions of any piece of electrical apparatus, applying to the design a most careful analysis as to its proportions, and who has satisfied himself that he has a most thoroughly designed piece of apparatus, will often be surprised and disappointed to find that some one else's design which has been less carefully calculated, or perhaps worked out entirely by the "cut and try" method, is, as far as its proportions are concerned, nearly or quite as good as his own carefully developed design. In other words, as Mr. Carter himself points out at the bottom of page 771 of his paper, quite large changes in the proportions of the various dimensions of the transformer designed, affect but slightly the final results obtained, and consequently more practical considerations make advisable, or even necessitate, marked departures from the most economical proportions.

While I have not had an opportunity to carefully study the paper, yet I have hastily checked up some of the proportions found based on Mr. Carter's study with actual transformers of which I know the proportions, and find that these transformers had dimensions differing in their relation some 200 or 300 per cent. from those which the paper showed to be the most econom-

ical, yet these transformers are even more efficient (considering the value of the material in them) than the transformer which Mr. Carter develops to illustrate his formulæ. In other words, considerations which are in one sense purely details of shop practice or skill in workmanship, may modify the results to such an extent as to entirely outweigh the consideration of best theoretical proportions.

While I am speaking, I trust I may be pardoned for answering one of the more practical questions brought up by Mr. Jackson. I refer to his question as to the value of the use of oil in transformers. This is a question which has always been a subject of much discussion, and is one which sometimes is almost as hard for the manufacturer to settle as for the users of the transformers to decide upon. Many factors enter to determine one's decision in this matter, but in general it can be safely said that the use of oil transformers has vastly increased in the past few years.

The question as to just how much the capacity of the transformer can be safely increased by the use of oil is not one subject to direct answer, because the value of oil varies so much with different designs of transformers. Of course, we all understand that oil acts in a double manner in increasing the capacity of the transformer; first, by excluding air, and thereby preventing oxidation of the insulating material, and consequently increasing the temperature at which the transformer may be safely operated, and second, and more important, it acts as a heat conducting medium to carry the heat generated in the transformer to the transformer case. The space between the transformer and the case is small, and the air within this space consequently is too confined to allow any great amount of circulation, and therefore it acts as a poor conductor of heat. Oil will move much more freely under a slight difference of temperature, or, perhaps more correctly speaking, its specific heat is so much greater than that of air, that even with a slow motion it acts much more efficiently in conveying heat to the enclosing case. Just how much however oil will increase the safe capacity of a transformer can only be answered for each size and design of transformer independently. A small transformer may not have its capacity appreciably increased by its use, whereas a large one lacking in radiating surface on the coils and iron, but having a large exposed surface in its case, may easily have its capacity doubled, or even trebled by the use of oil.

THE PRESIDENT:—While we are indebted to the author for presenting to us propositions concerning the efficiency of a transformer, which are capable of analysis and application, it seems important to bear in mind that these propositions are necessarily limited by the conditions stated in their enunciation, and that many other conditions must be taken into account in

the design and operation of commercial transformers. The deductions from these propositions probably require considerable modification to suit different conditions of practice. Cost, size, weight, excitation current, magnetic leakage, regulation and heating, are some of the factors which must be included in the design of a good commercial transformer, and the relation between these factors is so complex as to elude a general mathematical formula of design. Consequently, while I do not wish to undervalue the rules given in the paper for determining the design of the most efficient transformer, it seems important to bear in mind that the question of efficiency is not the only one which must be considered in practice, and that after all such rules have been taken into account, the method of trial and error must always be depended upon in arriving at the best design of a transformer at the present time.

MR. PHILIPPO TORCHIO:—Mr. President, one point which has not received attention in the discussion is on page 761, where the author alludes to the condition of work for which the transformer is designed. It is very important for the central station manager to get a transformer that will be most efficient under its special conditions of use. The author has in this diagram the daily load in amperes and also the ohmic transformer losses, proportional to the square of the amperes. The whole day ohmic losses represent a smaller or larger proportion of the total transformer losses according to the use of the transformer. If the transformer is going to operate at full load, say one hour a day, and be idle for 23 hours, the ohmic losses will be comparatively small; but if it operates 24 hours a day at full load, these losses will be large. Therefore, from the central station point of view, one transformer designed for a certain condition will not do as well under another condition. I think that this point of the load factor of the transformer is worth considering in the design of transformers. Of course, besides the condition of regulation, the cost of the transformer should also be included in the problem, together with the cost of energy in each special case, as the central station can afford to increase investment cost when the coal and running expenses are high; but when these are low, as, for instance, in water-power plants, the inducement is to reduce investment cost. I think this point of the transformer load factor brought out in the paper is very important for the design of the most economical transformer for actual operation.

MR. F. V. HENSHAW:—I think the last speaker has brought out a very salient point there in practical work, because I should rather imagine that most transformers are sold on their no-load consumption of energy, because if you get a transformer which wastes very little energy when the secondary is open, you will meet the conditions that probably at least nine-tenths of the central stations have, and for that reason it seems a little signifi-

cant to me that the author finds the Westinghouse transformer has half as much iron as the "most economical design." While I have not gone into the thing very deeply, it seems to indicate that the author is going for full load efficiencies, rather than minimum wasted energy at no load, and in that case it is somewhat questionable whether the formulæ which we have every reason to be thankful to the author for putting in such good shape, could be followed in practical work with entire success. It seems to me rather, from the practical point of view of selling transformers to central stations, that we ought to have one that would run with very small light load losses and very small hysteresis losses, while at full load the ohmic losses might be as high as they could possibly be made without burning up the insulation. I do not know whether that is treating it very scientifically, but it seems to me it is rather a practical way of looking at it.

MR. JAMES HAMBLET:—Mr. President, I have been very much interested in the discussion this evening. I think it has been truly valuable in one sense more than in any other. Many of us present noticed two or three years ago quite an undercurrent of criticism unfavorable to the extreme mathematicians of the INSTITUTE. There has been, I think some will remember, quite a complaint coming to the surface, and a great deal of talk was made that we had too much mathematics. I want to congratulate those present, Mr. President, on the fact that we have seen evidence to-night of some harmony of feeling between the extreme mathematicians and the practical engineers. I move we adjourn.

[The motion was carried and the meeting adjourned.]

DIED.

SMITH:—At Baltimore, Md., December 24, 1898. Captain Frederick H. Smith, City Engineer, of Baltimore, who was born in Pittsburg, Penn., November 10th, 1839, and elected an Associate Member of the INSTITUTE November 13, 1889. He had been in failing health for some time, but pneumonia was the immediate cause of his death. The early work of Captain Smith was with the Louisville and Nashville Railroad, from which he entered the service of the Southern Confederacy, continuing through the war. Latterly his work has been in bridge engineering in South America and this country. His leisure was largely devoted to scientific research. He was a gallant soldier, a bright and vigorous thinker, a deep student, a faithful friend, and a devoted husband and father.

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